Theory of Entropicity (ToE)

Living Review Letters Series (ToE LRLS) — Letter III

From
Information Geometry
to
Information Gravity

Information Geometry as the Origin of Einstein's Gravity:

Correspondence of the Obidi Action and the Einstein–Hilbert Action in the Theory of Entropicity (ToE)

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Abstract

This monograph establishes, with full mathematical and conceptual rigor, the deep structural correspondence between the Obidi Action of the Theory of Entropicity (ToE) and the Einstein–Hilbert Action of General Relativity (GR). We demonstrate that information geometry — comprising the Fisher–Rao metric of classical statistical distinguishability, the Fubini–Study metric of quantum state distinguishability, and the Amari–Čencov α-connections — constitutes the hidden geometric substratum from which gravitational dynamics emerge. The Obidi Action, the variational centerpiece of ToE, accomplishes for the entropic field precisely what the Einstein–Hilbert Action accomplishes for spacetime curvature: it converts a geometric manifold into a dynamical physical arena. We trace the complete logical and mathematical pipeline — from the ontological entropy field S(Λ) through the Hybrid Metric-Affine Space (HMAS), through the α–q constitutive constraint linking Rényi–Tsallis functional deformation to affine geometric asymmetry, and through the emergence of the Master Entropic Equation (MEE) — ultimately recovering Einstein's field equations as the low-gradient, near-equilibrium limit of the entropic field equations. The Einstein–Hilbert Action is thus subsumed within the Obidi Action as a special case, establishing gravity not as a fundamental geometric postulate but as an emergent consequence of information-geometric dynamics. We further examine the Vuli–Ndlela Integral as the entropy-weighted path-integral formulation that introduces irreversibility and the arrow of time into the framework. Connections to the broader landscape of entropic gravity — Jacobson's thermodynamic derivation, Verlinde's emergent gravity, and Bianconi's Gravity from Entropy — are discussed within the universal ToE hierarchy. The paper concludes with a table of structural correspondences and experimental pathways that distinguish ToE from its predecessors.

While gradient-dependent and disformal metric deformations are known, we uniquely identify the entropy field Sas the canonical driver and fix a rank-one sign-flip that produces a Lorentzian metric whose causal structure is aligned with entropy flow.

We prove global emergence theorems (nondegeneracy, foliation, global time function) and derive an explicit curvature correction $\Delta_{\mathrm{Obidi}}[S, G]$*, then show that, under coarse-graining, the information-geometric Obidi action reproduces the Einstein–Hilbert action in the IR.*

A central result of this work is the demonstration that Fisher–Rao information geometry, constrained by Čencov's theorem to remain strictly positive-definite and Markov-invariant, cannot by itself generate the Lorentzian structure required for physical spacetime. This establishes a fundamental obstruction: ***any program that seeks to derive General Relativity from information geometry must break Čencov invariance.*** *The Theory of Entropicity (ToE) resolves this obstruction through the* ***Obidi Transformation****, an entropy-gradient–driven, rank-one disformal deformation that relaxes the invariance conditions of Čencov's theorem and converts the Fisher–Rao metric into the Lorentzian* ***Obidi Metric****. This controlled "Čencov breaking" is shown to be the minimal and necessary mechanism by which information geometry acquires causal structure, a distinguished temporal direction, and the correct signature for emergent Einsteinian spacetime. In this way, ToE provides a logically complete pathway from entropic first principles to the Einstein Field Equations.*

Author Note:

Most of the mathematical details of these expositions are available in **Section 9** and the **Appendices**, and the [desirous] reader is hereby encouraged to refer to them for a full understanding of the **novel logic, machinery** and **mathematical theory** employed in this **present Letter III of the Theory of Entropicity (ToE) Living Review Letters Series (ToE LRLS).**

Keywords:

Theory of Entropicity (ToE); Obidi Action; Obidi Transformation; Obidi Metric; Obidi Relativistic Reduction Theorem; Einstein–Hilbert Action; Information Gravity; Information Geometry; Fisher–Rao metric; Fubini–Study metric; Bures metric; Amari–Čencov α-connections; Rényi–Tsallis entropy; Master Entropic Equation; Vuli–Ndlela Integral; Emergent Gravity; Entropic Field; General Relativity; Levi–Civita connection; Hybrid Metric-Affine Space (HMAS); Entropic Cosmological Constant.

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PART I

Foundations and Conceptual Architecture

1. Introduction: The Geometry Beneath Gravity

The history of fundamental physics is, at its core, a history of geometric revelations. Isaac Newton described gravity as a force propagating across a fixed Euclidean stage. Albert Einstein, in his Theory of General Relativity (GR), performed a conceptual inversion of remarkable depth: gravity is not a force at all, but the curvature of the spacetime manifold itself, sourced by mass-energy and governed by the Einstein–Hilbert Action. In Einstein's vision, the universe does not merely contain geometry — the universe is geometry.

Yet Einstein's framework, brilliant as it is, takes spacetime geometry as a primordial given. The metric tensor $g_{\mu\nu}$ and its associated Levi–Civita connection are fundamental objects from which everything else descends. The question that Einstein's framework leaves unanswered — and that the Theory of Entropicity (ToE) now addresses — is: from what does geometry itself emerge?

The Theory of Entropicity (ToE), first formulated and developed by John Onimisi Obidi beginning in 2025, provides a radical and mathematically precise answer: geometry, gravity, time, motion, and matter all emerge from the dynamics of a single ontological field — the entropy field S(x). This is not entropy in the familiar Boltzmann sense of a statistical byproduct of many-body disorder. In ToE, entropy is an active, continuous, dynamic scalar field that permeates all of existence, whose gradients generate curvature, whose fluxes produce motion, and whose conservation laws encode causality and the arrow of time.

At the heart of ToE lies the **Obidi Action** — a variational principle for the entropy field that occupies precisely the same structural role in ToE as the Einstein–Hilbert Action occupies in General Relativity. Both actions are integral functionals over a four-dimensional spacetime manifold. Both yield, upon variation, second-order field equations that govern the co-evolution of fields and geometry. But where the Einstein–Hilbert Action treats spacetime curvature as fundamental, the Obidi Action treats entropic curvature as fundamental, and derives spacetime curvature as emergent.

This monograph — Letter III in the Theory of Entropicity Living Review Letters Series (**ToE LRLS**) — presents a comprehensive, monograph-grade exposition of the correspondence between the Obidi Action and the Einstein–Hilbert Action. We trace the complete mathematical pipeline from information geometry to physical gravity, establishing the principle that information geometry is the origin of gravity. The paper proceeds as follows:

- Part I establishes the conceptual architecture: the status of entropy as a fundamental field, the structure of the Obidi Action and Master Entropic Equation, and the logical motivation for an information-geometric foundation for physics.
- Part II develops the mathematical machinery: the Hybrid Metric-Affine Space, the Fisher–Rao and Fubini–Study metrics, the Amari–Čencov α-connections, the Rényi–Tsallis generalization, and the constitutive α–q correspondence.
- Part III establishes the formal correspondence between the Obidi Action and the Einstein–Hilbert Action, deriving Einstein's field equations as a limiting case of the entropic field equations.
- Part IV situates ToE within the broader landscape of entropic gravity and discusses experimental consequences and future directions.

1.1 The Central Claim

The central claim of this paper may be stated as the following correspondence principle:

Obidi Action : Entropic Field ≡ Einstein–Hilbert Action : Spacetime Curvature

More precisely: the Einstein–Hilbert Action and its resulting field equations are recovered, with mathematical exactness, from the Obidi Action in the limit where entropy gradients are negligible, the entropy field is near equilibrium, and quantum corrections are suppressed. The Obidi Action thus does not merely analogize the Einstein–Hilbert Action — it contains it as a special case.

1.2 Scope and Notation

Throughout this paper, we adopt natural units where kB = 1 unless otherwise stated for clarity. Greek indices μ, ν run over spacetime coordinates 0, 1, 2, 3. The metric signature is (−, +, +, +). The entropy field S(x) is a real-valued scalar field on a Lorentzian manifold (M, gμν). The symbol ∇μ denotes the covariant derivative with respect to the Levi–Civita connection unless otherwise specified. Angle brackets ⟨·⟩ denote ensemble or cosmological averaging. The symbol ⊂ denotes set-theoretic embedding in the hierarchy of theories.

2. Entropy as the Fundamental Ontological Field

Classical thermodynamics, statistical mechanics, and information theory have each developed their own interpretations of entropy, all of which share a common limitation: they treat entropy as a derived or secondary quantity — a measure of disorder, uncertainty, or missing information — rather than as a primary constituent of physical reality. The Theory of Entropicity radically inverts this perspective.

2.1 The Ontological Declaration

ToE makes the following foundational ontological declaration: Entropy S(Λ) is not a statistical descriptor of an underlying reality. It is that underlying reality. Geometry, matter, fields, and forces are not the primordial furniture of the universe — they are emergent manifestations of a more fundamental entropic substratum.

This declaration is philosophically comparable in ambition to Einstein's declaration that gravity is not a force but the curvature of spacetime. Where Einstein's revolution was geometrical, Obidi's revolution is entropic. And as Einstein's revolution did not deny the existence of forces but re-described them as curvature, Obidi's revolution does not deny the existence of geometry but re-describes it as an emergent projection of entropy dynamics.

The universe, in ToE's ontology, is an entropic manifold — a structured arena in which the entropy field continuously rearranges itself, and in doing so generates the apparent structures of spacetime, matter, and causality that conventional physics takes as given. This philosophical orientation is captured in the term **ontodynamics**, which ToE uses to describe the study of how existence itself evolves through entropic dynamics.

2.2 The Entropy Field S(x)

The entropy field S(Λ) is a real-valued scalar field defined on a differentiable Lorentzian manifold M. Unlike conventional fields in quantum field theory, which represent excitations of a pre-existing spacetime background, S(Λ) is the field from which the background itself emerges. It is a pre-geometric field: it exists prior to and independently of the spacetime metric, and it is the dynamics of S(Λ) that generate the metric as an emergent structure.

In the classical (large-scale) regime, S(Λ) admits a description in terms of smooth gradients and a well-defined local entropy density. In the quantum regime, S(Λ) must be treated as an operator-valued field, and its fluctuations generate quantum corrections to the emergent geometry. The Vuli–Ndlela Integral (discussed in Section 5) provides the quantum formulation.

[NB: From now on, we take it for granted that the reader already understands the Obidi pre-geometric transformation Λ→x, such that S(Λ) →S(x).]

The crucial insight is that S(x) is not a function defined on spacetime — it is the generator of spacetime. The gradients of S(x) produce the curvature and connection that conventional physics identifies as the structure of spacetime. The cosmological constant emerges from the macroscopic variance of those gradients. The arrow of time emerges from the irreversibility of entropic flow.

2.3 Speed of Light as the Entropic Speed Limit

One of the most striking reinterpretations [and derivations] in ToE concerns the universal constant c [the conventional speed of light]. In Special Relativity, c is postulated as the invariant speed of light in vacuum. In ToE, c is reinterpreted as the maximum rate at which the entropy field can reorganize energy and information:

$$c \equiv \text{max rate of entropic rearrangement of } S(x) \quad (2.1)$$

This is not merely a verbal restatement. It implies that the causal structure of spacetime — light cones, simultaneity, time dilation, length contraction, mass increase — are all consequences of the constraint that entropic reorganization cannot exceed rate c. In this sense, Lorentz invariance is not a postulate about geometry but a theorem about the dynamics of the entropic field.

This reinterpretation has been made precise in Obidi's derivation of Einstein's second postulate from entropic principles, and in the derivation of time dilation and length contraction from the **Entropic Resistance Principle (ERP)** and **Entropic Accounting Principle (EAP)** of ToE.

3. The Obidi Action: ToE's Variational Principle

The cornerstone of the Theory of Entropicity (ToE) is a variational principle for the entropy field — the Obidi Action. This action occupies the same structural position in ToE that the Einstein–Hilbert Action occupies in General Relativity, and that the action functional occupies in any Lagrangian field theory. It is not merely a technical tool; it is the mathematical statement of the theory's deepest physical principle.

3.1 The Obidi Action: Full Form

The Obidi Action in its full, general form is:

$$A_{\text{ToE}}[S; g] = \int_M d^4 x \sqrt{-g}\, [\chi(\nabla_\mu S)(\nabla^\mu S) - V(S) + J(x, S)] \qquad (3.1)$$

where:

- $S(x)$ is the ontological scalar entropy field, the fundamental degree of freedom of ToE;
- $g_{\mu\nu}$ is the spacetime metric (itself emergent from S(x) in the full theory, but treated as background in the semiclassical approximation);
- $\sqrt{-g}\, d^4x$ is the covariant volume element, ensuring diffeomorphism invariance;
- χ is the kinetic coupling constant, encoding the "stiffness" of entropic fluctuations;
- $(\nabla_\mu S)(\nabla^\mu S) = g^{\mu\nu}(\nabla_\mu S)(\nabla_\nu S)$ is the kinetic term, representing the energetic cost of entropy gradients; (**Note:** Since $\nabla^\mu S = g^{\mu\nu}\nabla_\nu S$, we have: $\big(\nabla_\mu S)(\nabla^\mu S) = (\nabla_\mu S)\, g^{\mu\nu}(\nabla_\nu S)\big) =$ $g^{\mu\nu}(\nabla_\mu S)\, (\nabla_\nu S)$;
- $V(S)$is the entropic potential, encoding the self-interaction of the entropy field and generating the effective mass and cosmological terms;
- $J(x, S)$ is the source/coupling term, encoding interactions of the entropy field with matter and information.

The mathematical structure of (3.1) is immediately recognizable as a scalar field theory on a curved spacetime manifold. Comparison with the Klein–Gordon action, the scalar field action in QFT, and the dilaton action in string theory reveals the same formal structure — but with the crucial difference that S(x) is ontologically primary, not a field defined on a pre-existing spacetime.

3.2 The Master Entropic Equation

Variation of the Obidi Action with respect to $S(x)$ — holding the metric fixed — yields the Master Entropic Equation (MEE), which governs the dynamics of the entropy field:

$$\kappa_S \nabla_\mu \nabla^\mu S - \frac{dV}{dS} + \Lambda_S(S, \nabla S, g) = 0 \qquad (3.2)$$

or, in its more compact form:

$$\Box_g S - \frac{1}{\kappa_S}\frac{dV}{dS} = J(x) \qquad (3.3)$$

Where $\Box_g = g^{\mu\nu}(\nabla_\mu)\,(\nabla_\nu)$ is the covariant d'Alembertian. This equation is the entropic analogue of the Klein–Gordon equation, the Maxwell equations, and ultimately Einstein's field equations — it is the universal law governing the evolution of the entropic substrate from which all physical phenomena emerge.

The MEE has solutions corresponding to:

- Entropic geodesics: the paths followed by freely-falling bodies in the emergent spacetime, arising from the gradient flow of S(x);
- Entropy potential equation: the equation governing the shape of the V(S) landscape and its relation to curvature;
- Emergent gravity equations: in the appropriate limit, the MEE reduces to Einstein's field equations (as demonstrated in Part III).

3.3 The Entropic Current and Conservation Law

From the Obidi Action, one derives a natural entropic current:

$$J^\mu = \eta\, \nabla^\mu S (\eta > 0) \qquad (3.4)$$

where $\eta > 0$ is the entropic coupling constant. The conservation of this current:

$$\nabla_\mu J^\mu = 0 \qquad (3.5)$$

is the entropic analogue of the conservation of electric charge in electrodynamics, or of the conservation of energy-momentum in GR. It enforces that entropy flux is globally preserved across spacetime, and is the source of the Lagrange multiplier structure that generates the auxiliary G-field and emergent cosmological constant discussed in Section 7.

3.4 The Entropic Stress-Energy Tensor

Variation of the Obidi Action with respect to the metric $g_{\mu\nu}$ yields the entropic stress-energy tensor:

$$T_{\mu\nu}^{(S)} = \nabla_\mu S\,\nabla_\nu S - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha S)(\nabla^\alpha S) + g_{\mu\nu}V(S) - g_{\mu\nu}J(x)S \qquad (3.6)$$

This tensor sources the emergent gravitational field equations (3.13) below, playing precisely the role that the matter stress-energy tensor $T_{\mu\nu}^{(m)}$ plays in Einstein's GR. The identification $T_{\mu\nu}^{(S)} \leftrightarrow (1/8\pi G)G_{\mu\nu}$ (Einstein tensor) is exact in the low-gradient limit, as we shall demonstrate in Part III.

4. Information Geometry as the Mathematical Foundation

The Theory of Entropicity (ToE) does not merely invoke information-theoretic language metaphorically. It rests on a precise and rigorous mathematical framework — information geometry — that provides the geometric foundation from which physical spacetime emerges. This section reviews the key structures: the Fisher–Rao metric, the Fubini–Study metric, and the Amari–Čencov α-connections.

4.1 Statistical Manifolds and the Fisher–Rao Metric

Information geometry, developed principally by Shun-ichi Amari and C.R. Rao (building on R.A. Fisher's original work), studies the differential geometry of families of probability distributions. For a parametric family $p(x \mid \theta)$ with parameters $\theta = (\theta_1, \ldots, \theta_{\mathrm{n}})$, the Fisher–Rao metric is defined as:

$$g_{ij}^{(\mathrm{FR})} = \int p(x \mid \theta) \frac{\partial}{\partial \theta^i} \ln\, p(x \mid \theta) \frac{\partial}{\partial \theta^j} \ln\, p(x \mid \theta)\, dx \qquad (4.1)$$

This metric quantifies the distinguishability between neighboring probability distributions. Two distributions separated by a unit Fisher–Rao distance can be reliably distinguished by statistical inference. It is the canonical Riemannian metric on the statistical manifold, and it is positive-definite and covariant under reparameterization.

In the ToE framework, the entropy field $S(x)$ generates a family of probability distributions $p(\cdot\, |\theta; S)$ over the space of possible physical configurations. The Fisher–Rao metric thereby inherits a dependence on $S(x)$ and becomes a field on the entropic manifold. This is the first step in the bridge from information geometry to physical geometry.

4.2 The Fubini–Study Metric and Quantum Sector

In the quantum domain, the natural counterpart to the Fisher–Rao metric is the Fubini–Study metric on the complex projective Hilbert space $CP(H)$:

$$ds_{\mathrm{FS}}^2 = 4(1 - |\langle \psi \mid \psi + d\psi \rangle|^2) \simeq g_{\mathrm{FS}}(S)(dS)^2 \qquad (4.2)$$

Where $g_{\mathrm{FS}}(S) = {\partial^2}/{\partial^2 S}\, [\ln \mathrm{Z(S)}] = \frac{\partial^2 [\ln \mathrm{Z(S)}]}{\partial^2 \mathrm{S}}$ is the quantum Fisher information, which plays the role of the metric in the quantum statistical manifold. The Fubini–Study metric measures the distinguishability between pure quantum states — the geometric core of quantum mechanics.

In ToE, both metrics coexist on the entropic manifold, unified within the Hybrid Metric-Affine Space (HMAS). Classical physics "sees" the Fisher–Rao sector; quantum theory "sees" the Fubini–Study sector. ToE provides the unifying entropic geometry that contains both as limiting cases.

4.3 The Hybrid Metric-Affine Space (HMAS)

The **Hybrid Metric-Affine Space (HMAS)** is the geometric arena of the Theory of Entropicity (ToE). It is a metric-affine manifold in which the metric is assembled from both classical and quantum information-geometric contributions:

$$G_{\mu\nu}(S; g) = e^{\alpha(S)} g_{\mu\nu} + \lambda_Q g_{\mu\nu}^{(\mathrm{FS})}(S) \qquad (4.3)$$

where $\alpha(S) = S/k_B + O(S^2/{k_B}^2)$ encodes the classical (Fisher–Rao) entropic deformation of the background metric, and λ_Q controls the weight of the Fubini–Study (quantum) correction. The entropy-weighted metric $G_{\mu\nu}(S; g)$ continuously deforms from the background metric $g_{\mu\nu}$ as the entropy field departs from equilibrium.

The eigenvalues $\{\lambda_i\}$ of $G^{\mu\nu}$ with respect to $g^{\mu\nu}$ measure the local entropic distortion of geometry. Near equilibrium ($S \approx S_{eq}$, $\delta S = S - S_{eq}$ small), one finds:

$$\lambda_i(x; S) = 1 + \beta(x)\delta S(x) + O(\delta S^2) \qquad (4.4)$$

Where $\beta(x) = \alpha'(S_{eq}) + \lambda_Q g'_{FS}(S_{eq})$ encodes both classical and quantum deformation coefficients. **The spectral Kullback–Leibler divergence between $G_{\mu\nu}$ and the equilibrium metric provides the local relative-entropy density from which Bianconi's action emerges as a limiting case.**

4.4 The Amari–Čencov α-Connections: The Geometric Transformer

The Fisher–Rao and Fubini–Study metrics, while providing a notion of distance on the information manifold, do not by themselves determine how vectors or tensors are transported — they lack an affine connection. Amari and Čencov introduced a one-parameter family of dual affine connections that supply this missing structure:

$$\nabla^{(\alpha)} = \nabla^{(0)} + \frac{\alpha}{2} T \qquad (4.5)$$

where T is the third-order expectation tensor (the "skewness tensor") of the statistical model:

$$T_{ijk} = \int p(x \mid \theta) \frac{\partial \ln\, p}{\partial \theta^i} \frac{\partial \ln\, p}{\partial \theta^j} \frac{\partial \ln\, p}{\partial \theta^k} dx \qquad (4.6)$$

and $\nabla^{(0)}$ is the Levi–Civita connection of the Fisher–Rao metric. The parameter α interpolates continuously between the exponential connection (α = +1), the Levi–Civita connection (α = 0), and the mixture connection (α = −1).

The physical significance of [the Amari–Čencov] α [-connection] within the Theory of Entropicity (ToE) is profound:

- α = 0: reversible, equilibrium dynamics; information flow is symmetric; no arrow of time.
- α ≠ 0: irreversible, non-equilibrium dynamics; the manifold has an affine asymmetry that encodes the direction of entropy flow.
- α > 0: super-extensive regime; entropy diverges and drives expansion (cf. dark energy).
- α < 0: sub-extensive regime; entropy converges and drives localization (cf. gravitational collapse).

The Amari–Čencov α-connection is thus the geometric expression of the arrow of time. It is what makes the information manifold physically dynamical — not merely a static arena of statistical relationships, but an evolving structure that encodes the causal history of the universe.

Most importantly for the correspondence with GR: in the appropriate classical, large-scale limit where entropy gradients and quantum corrections are coarse-grained and the system is near equilibrium, the effective connection reduces to a torsion-free, metric-compatible connection — precisely the Levi–Civita connection of the emergent spacetime metric. This is the connection that underlies Einstein's field equations. Hence, the Levi–Civita connection of GR is not a fundamental geometric datum but an effective, emergent simplification of the entropic α-connection structure.

5. The Rényi–Tsallis Generalization and the α–q Constitutive Constraint

5.1 Non-Extensive Entropies

Standard Boltzmann–Gibbs entropy is extensive: the entropy of two independent systems equals the sum of their individual entropies. Real physical systems — those with long-range correlations, fractal structure, or multi-scale dynamics — violate this extensivity. The Rényi and Tsallis generalizations capture this:

Tsallis entropy:

$$S_q^{(\text{Tsallis})} = k_B \frac{1 - \sum_i p_i^q}{q - 1} \qquad (5.1)$$

Rényi entropy:

$$S_\alpha^{(\text{Rényi})} = \frac{k_B}{1 - \alpha} \ln \left(\sum_i p_i^\alpha \right) \qquad (5.2)$$

Both recover Boltzmann–Gibbs entropy as q → 1 (or α → 1). The deformation parameter q (or α in the Rényi case) measures the degree of non-extensivity: q > 1 implies super-extensivity with long-range correlations (relevant for self-gravitating systems, dark energy); q < 1 implies sub-extensivity with multi-fractal structure (relevant for systems with excluded volume or short-range cutoffs).

Within ToE, these non-extensive entropies are not separate theories but different sectors of the same universal entropic field theory. The deformation parameter q enters the shape of the entropic potential V(S) and determines the curvature of the information manifold.

5.2 The α–q Constitutive Constraint

A fundamental identity in information geometry links the non-extensivity parameter of Tsallis/Rényi entropy to the affine asymmetry parameter of the Amari–Čencov connections. This arises by comparing the Tsallis q-divergence:

$$D_q(p \parallel r) = \frac{1}{q - 1}\left(1 - \sum_i p_i^q \, r_i^{1-q}\right) \qquad (5.3)$$

with the [**Kullback-Leibler (KL) or Araki-Umegaki (AU)**] α-divergence:

$$D_\alpha(p \parallel r) = \frac{4}{1-\alpha^2}\left(1 - \sum_i p_i^{\frac{1-\alpha}{2}} r_i^{\frac{1+\alpha}{2}}\right) \qquad (5.4)$$

Requiring that both divergences produce the same Fisher–Rao metric at equilibrium (i.e., both expand to the same quadratic form to second order in perturbations) yields the identification:

$$\alpha = 2(1-q) \qquad (5.5)$$

This is the α–q constitutive constraint of the Theory of Entropicity (ToE). Its physical significance cannot be overstated:

- It links functional non-extensivity (q, acting in probability space) to geometric affine asymmetry (α, acting in connection space). These are dual descriptions of the same underlying entropic deformation.
- For q = 1 (Boltzmann–Gibbs): α = 0, flat Fisher–Rao geometry, reversible dynamics, Levi–Civita connection — this is the regime of standard GR.
- For q > 1 (super-extensive): α < 0, divergent entropic flow — relevant for cosmological acceleration.
- For q < 1 (sub-extensive): α > 0, convergent entropic flow — relevant for strong gravity and localization.

Within the Theory of Entropicity (ToE), equation (5.5) is not merely a mathematical equivalence. It is embedded as a constitutive law within the Obidi Action, which enforces α = α(q) as a dynamical constraint rather than a choice. This transforms what was previously a "mathematical coincidence" in information geometry literature into a physical coupling principle.

5.3 The ToE Generalized Divergence Family

By varying the parameters (α, q) within the Theory of Entropicity (ToE) entropic potential, one obtains a one-parameter family of entropic field theories:

$$V_{\alpha,q}(S) \equiv \kappa\, D_{\alpha,q}(\rho_S \parallel \sigma_{S_{\mathrm{eq}}}) \qquad (5.6)$$

Each value of (α, q) defines a distinct "sector" of the Theory of Entropicity (ToE) with its own effective gravitational coupling and cosmological constant:

Sector / Regime	Description
$(\alpha, q) = (0, 1)$	Boltzmann–Gibbs limit; flat Fisher–Rao geometry; classical GR emerges
Quadratic expansion around Seq (Entropic Equilibrium)	Bianconi's Gravity from Entropy (special case of ToE)
$\alpha \to 0$, $(\alpha,q) \to (1,1)$, δS small	Full Bianconi limit with KL divergence structure
$(\alpha, q) \to (1, 1)$	Fisher–Shannon/von Neumann sector; quantum GR corrections
General $(\alpha, q) \neq (1,1)$	Full ToE hierarchy; Rényi–Tsallis entropic gravity
$\nabla S = 0$, $V'(S) = \text{const}$	Standard Einstein GR recovered exactly

Table 1: Hierarchy of entropic sectors within ToE. The full theory contains all sectors simultaneously.

PART II

The Mathematical Correspondence

6. The Einstein–Hilbert Action: Structure and Role

Before establishing the correspondence, it is useful to recall the structure of the Einstein–Hilbert Action and its role in Einstein's General Relativity (GR), so that the structural parallels with the Obidi Action can be made precise.

6.1 The Einstein–Hilbert Action

The Einstein–Hilbert Action is:

$$S_{EH}[g] = \frac{1}{16\pi G}\int d^4 x\sqrt{-g}\,R + S_M[\Phi, g] \ (6.1)$$

where:

- G is Newton's gravitational constant;
- $R = g^{\mu\nu}R_{\mu\nu}$is the Ricci scalar, the trace of the Ricci curvature tensor;
- $S_M[\Phi, g]$ is the matter action, coupling matter fields Φ to the metric gμν;
- $\sqrt{-g}\,d^4x$ is the invariant volume element.

Variation of SEH with respect to the (inverse) metric $g_{\mu\nu}$ yields the celebrated Einstein's field equations (EFE):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\,T_{\mu\nu}^{(m)} \ (6.2)$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the famous Einstein tensor, Λ is the [perennially problematic] cosmological constant, and $T_{\mu\nu}^{(m)}$ is the [ingenious] matter stress-energy tensor. **The Einstein–Hilbert Action thus converts a geometric degree of freedom — the metric $g_{\mu\nu}$ — into a dynamical quantity whose evolution is governed by the matter content.**

6.1.1 Scholium on the LHS Einstein Tensor of the Einstein Field Equations

As given earlier above, this is the **Einstein tensor** in **General Relativity (GR):**

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \ (6.2.1)$$

It is the central geometric object on the left-hand side of Einstein's beautiful field equations.

More generally, in its most familiar form, we write it as:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (6.2.2)$$

where:

$G_{\mu\nu}$ is the **Einstein tensor**, representing the curvature structure of spacetime.
$R_{\mu\nu}$ is the **Ricci curvature tensor**, measuring how volumes deform due to curvature.

$g_{\mu\nu}$ is the **metric tensor**, defining distances, intervals, angles, and causal structure in spacetime.

R is the **Ricci scalar**, the trace of the Ricci tensor:

$$R = g^{\mu\nu} R_{\mu\nu} \quad (6.2.3)$$

Thus, the expression

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad (6.2.4)$$

is not arbitrary. It is constructed so that its covariant divergence vanishes:

$$\nabla^{\mu} G_{\mu\nu} = 0 \quad (6.2.5)$$

This is crucial because it matches the conservation law for matter and energy:

$$\nabla^{\mu} T_{\mu\nu} = 0 \quad (6.2.6)$$

In other words, the Einstein tensor is the mathematically balanced curvature object that allows geometry to correspond consistently to matter-energy.

For the Theory of Entropicity (ToE), this is the object that ToE would need to recover, reinterpret, or generalize as it demands that gravity emerges not from spacetime curvature as fundamental, but from entropy/information geometry. In that case, therefore, ToE's task is to show how an entropic or information-geometric action yields an effective tensor that reduces to the Einstein tensor:

$$G_{\mu\nu} \quad (6.2.7)$$

in the classical General Relativity (GR) limit.

6.1.2 Scholium on the RHS Matter Stress-Energy Tensor of the Einstein Field Equations

The right side (RHS) of the Einstein field equations is the **matter-energy side** of the equation. It tells spacetime (on the LHS) what physical content is present (on the RHS).

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (6.2.8)$$

The term:

$$T_{\mu\nu} \quad (6.2.9)$$

is called the **stress-energy tensor** or **energy-momentum tensor**.

It represents the full physical content of matter, radiation, pressure, momentum, and stress. It is not merely "mass." In General Relativity, gravity is sourced by the entire distribution of energy and momentum, not by mass alone.

The coefficient:

$$\frac{8\pi G}{c^4} \quad (6.2.10)$$

is the **gravitational coupling constant** in Einstein's equation. It tells how strongly matter-energy produces curvature.

Here:

$$G \quad (6.2.11)$$

is Newton's gravitational constant. It controls the strength of gravity in the classical Newtonian limit.

$$c \quad (6.2.12)$$

is the speed of light. This factor c^4 appears because Einstein's equation relates geometry, matter, energy, and relativistic units in a way consistent with special relativity.

So, to state it in plain language:

$$\frac{8\pi G}{c^4} T_{\mu\nu} \quad (6.2.13)$$

means:

The matter-energy content of the universe, multiplied by the correct gravitational conversion factor, produces spacetime curvature.

More precisely, the stress-energy tensor contains several kinds of physical information.

The component:

$$T_{00} \quad (6.2.14)$$

represents **energy density**. This is the part most closely related to ordinary mass-energy.

The components:

$$T_{0i} \quad (6.2.15)$$

represent **momentum density** or energy flow.

The components:

$$T_{ij} \quad (6.2.16)$$

represent **pressure and internal stresses**.

This is why General Relativity says that pressure can also gravitate. A star, a radiation field, a black hole environment, or the early universe cannot be described by mass density alone. Their pressure, momentum flow, and stress also contribute to gravity.

Therefore, the left side:

$$G_{\mu\nu} \quad (6.2.17)$$

describes **geometry and curvature**, while the right side:

$$\frac{8\pi G}{c^4} T_{\mu\nu} \quad (6.2.18)$$

describes **matter, energy, pressure, momentum, and stress**.

A compact interpretation is thus:

$$\text{Curvature} = \text{Matter-energy source} \quad (6.2.19)$$

or more physically:

spacetime curvature is determined by the distribution and flow of energy, momentum, pressure, and stress.

If the cosmological constant is included, the equation is then often written as:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \ (6.2.20)$$

In that form, the right-hand side remains the ordinary matter-energy tensor, while $\Lambda g_{\mu\nu}$represents the cosmological constant contribution, often associated with vacuum energy or dark-energy-like behavior.

6.1.3 Scholium on the RHS of the Einstein Field Equations and Its Implication for the Theory of Entropicity (ToE)

The Right-Hand Side of Einstein's Equation and Its Implication for the Theory of Entropicity

The right-hand side of Einstein's field equation is one of the most important entry points for clarifying what the Theory of Entropicity must accomplish if it is to be more than a reinterpretation of gravitational geometry. Einstein's field equation is commonly written as

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \ (6.2.21)$$

The left-hand side, $(G_{\mu\nu})$, is the Einstein tensor. It describes the curvature structure of spacetime. The right-hand side, $\left(\frac{8\pi G}{c^4} T_{\mu\nu}\right)$, is the source term. It describes the matter-energy content that gives rise to curvature. Therefore, the full equation does not merely say that geometry exists. It says that geometry is dynamically related to matter, energy, pressure, momentum, stress, and radiation. This is the decisive implication for the Theory of Entropicity: ToE must not merely explain curvature; it must also explain the source of curvature.

A weak formulation of ToE would only claim that entropy produces geometry. Such a claim would address the left-hand side of Einstein's equation, but it would leave the right-hand side untouched. A stronger and more complete formulation of ToE must go further. It must show that what General Relativity calls the stress-energy tensor is itself an emergent expression of entropy, information, constraint, and entropic flow. In that deeper formulation, the Einstein equation would no longer be read as a relation between two independent realities, namely geometry on one side and matter-energy on the other. Rather, it would be read as a correspondence between two macroscopic manifestations of one deeper substrate: entropy as curvature on the left and entropy as organized matter-energy on the right.

This is where the right-hand side becomes central to the philosophical and mathematical ambition of ToE. In General Relativity, ($T_{\mu\nu}$) is not merely a symbol for mass. It is a rank-two tensor containing the total physical source structure of the system. Its temporal component represents energy density. Its mixed temporal-spatial components represent momentum density and energy flux. Its spatial components represent pressure, shear, and internal stress. This means that the source of gravity in General Relativity is not simply matter in the ordinary Newtonian sense. It is a complete energetic and dynamical distribution. Gravity responds not only to mass, but also to pressure, momentum, radiation, and stress.

For the Theory of Entropicity, this has a profound implication. Entropy cannot be treated merely as a scalar quantity of disorder. If entropy is to become the foundation of gravitational dynamics, it must be capable of producing a tensorial source structure. The theory must therefore move from entropy as a scalar measure to entropy as a field-theoretic and information-geometric structure capable of generating ($T_{\mu\nu}$). A scalar entropy field may be foundational, but by itself it cannot simply replace the stress-energy tensor. It must be differentiated, constrained, coupled, and varied through an action principle so that a genuine tensor emerges.

This is why the Obidi Action becomes essential. The Obidi Action must not function only as a formal analogue of the Einstein–Hilbert action. It must become the deeper variational principle from which both sides of Einstein's equation can be recovered in the appropriate classical limit. On the geometric side, the Obidi Action must yield an effective curvature tensor corresponding to ($G_{\mu\nu}$). On the source side, it must yield an entropic stress-energy tensor corresponding to ($T_{\mu\nu}$). Only then can ToE claim to explain Einstein gravity from entropy rather than merely restating Einstein gravity in entropic language.

In this framework, matter must be understood as entropy in a constrained and stabilized form. Mass is not treated as an irreducible primitive. Rather, mass becomes the manifestation of stable internal entropic organization. A material object possesses mass because its internal degrees of freedom are not freely dispersed. They are constrained into a persistent structure. That constraint stores information, resists arbitrary rearrangement, and presents itself externally as matter-energy. The right-hand side of Einstein's equation therefore becomes, in ToE, the macroscopic tensorial expression of local entropic constraint.

This interpretation makes the connection with information entropy especially significant. If local information entropy decreases inside matter, that decrease need not contradict the broader thermodynamic arrow. In the ToE picture, the universe may possess a global entropic direction while still allowing local pockets of reduced information entropy. Such local reductions are precisely what make structure possible. Stars, atoms, molecules, fields, biological systems, black holes, and observers all require local constraint. They are not violations of entropy; they are organized regions sustained within a larger entropic evolution.

In this sense, [the University of Portsmouth's] **Dr. Melvin Vopson's proposition that information entropy decreases** in connection with gravity can be absorbed into ToE as a local phenomenon rather than treated as a contradiction. **The ToE interpretation is not that global entropy simply decreases. Rather, local information entropy may decrease where matter [and consciousness] forms, because matter is a region of constrained information.** The geometric response to that constraint appears as curvature. Gravity then becomes the coupling between local information constraint[s] and global entropic geometry. Thus, the **Theory of Entropicity (ToE) chain of reasoning then becomes: local information entropy decreases into matter [and/or mass]; matter expresses itself as [mass and /or] stress-energy; [mass and/or] stress-energy induces curvature; curvature manifests as gravity.**

This allows ToE to distinguish between two levels of entropy without dividing reality into two unrelated substances. There is global geometric entropy, which governs the large-scale curvature, temporal direction, and dynamical evolution of the universe. There is also local informational entropy, which may decrease where matter, structure, and stable physical identity emerge. The two are not enemies or opponents. **They are complementary levels of one entropic universe**. Local informational decrease produces constrained matter; global geometric entropy supplies the field background through which that constrained matter produces curvature.

The stress-energy tensor can therefore be reinterpreted as the tensor of entropic embodiment. **Its (T_{00})** component corresponds to **local entropic energy density**: the stored capacity of constrained information to act as energy. **Its (T_{0i})** components correspond to **entropic momentum flux**: the **directional transport of energy and information through the field**. Its (T_{ij}) components correspond to **entropic pressure and internal [entropic] resistance**: the way **constrained entropy pushes, stresses, and resists deformation [re-ordering/redistribution/re-organization/rearrangement,**

etc.]. This is especially important because General Relativity already teaches that pressure gravitates [that is, it is not only mass/matter perse that creates/generates gravity, but pressure, radiation, etc., infact all the components of the stress-energy tensor $T_{\mu\nu}$]. The Theory of Entropicity (ToE) thus takes this as a direct and valid clue that gravity is not merely a response to inert mass, but to the full thermodynamic and informational condition of a system, all of which arise from [and as a result of] entropy.

However, we must make ToE mathematically cautious and precise at this point. It would be incorrect to write simply that ($T_{\mu\nu} = S$), or ($T_{\mu\nu} = \Lambda$), if (S) or (Λ) is only a **Scalar Entropy Field (SEF).** The correct and logical move would be to construct the stress-energy tensor from the Entropic Field (EF) of the Theory of Entropicity (ToE) itself, employing its gradients, its currents, its potentials, and its constraints [which help generate and construct the appropriate, necessary, and requisite tensorial objects]. A rigorous ToE formulation must therefore take a form closer to:

$$-\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{source}}^{(\text{ToE})}}{\delta g^{\mu\nu}}, \quad (6.2.22)$$

where ($S_{\text{source}}^{(\text{ToE})}$) is the source sector of the Obidi Action. In such a formulation, ($T_{\mu\nu}^{(\text{ToE})}$) is not inserted by hand. It is obtained by varying the entropic matter-source action with respect to the metric. This is the standard route by which field theories generate stress-energy tensors, but ToE gives the source sector a new interpretation: **the source is not primitive matter, but constrained entropy and information structure [constrained entropy and flowing entropy, entropy gradients, current, etc.].**

A possible entropic source sector would involve quantities such as the (1) **entropy field (Λ)**, (2) its **gradient ($\nabla_\mu \Lambda$)**, (3) an **entropy-current ($J_\mu^{(S)}$),** (4) an **entropic potential ($V(\Lambda)$),** and (5) [**entropic**] **information-constraint terms** encoding the (a) **local reduction of informational freedom [Obidi's utilization of [Dr. Melvin] Vopson's information dynamics (infodynamics)]**. From such objects, tensorial combinations can be formed, for example:

$$\nabla_\mu \Lambda \nabla_\nu \Lambda \quad (6.2.23),$$

$$J_\mu^{(S)} J_\nu^{(S)} \quad (6.2.24), \text{ and}$$

$$g_{\mu\nu} V(\Lambda). \quad (6.2.25)$$

These terms show how a scalar or current-based entropic theory can generate a rank-two tensor. The stress-energy tensor is then not entropy itself, but the tensorial expression of how entropy is distributed, constrained, flowing, stored, and resisted.

6.1.3.1 Scholium on the Above Last Statement on How the Theory of Entropicity (ToE) Must Recover the Einstein Stress-Energy Tensor of General Relativity (GR)

The significance of this formulation is that it prevents the Theory of Entropicity (ToE) from collapsing into a mathematically inadequate identification of entropy with the stress-energy tensor. In General Relativity, the stress-energy tensor is not a mere scalar quantity, nor is it reducible to mass alone. It is the full tensorial representation of **energy density**, **momentum flux**, **pressure**, **shear**, **stress**, and **internal dynamical resistance**. **Therefore, if entropy is to be treated as fundamental in the Theory of Entropicity (ToE), it cannot simply be placed on the right-hand side of Einstein's field equations as an undifferentiated scalar. Entropy must therefore first acquire tensorial embodiment through its distribution, gradients, currents, constraints, storage modes, and resistance structures**. **The stress-energy tensor is therefore not [scalar] entropy itself, but the macroscopic tensorial expression of entropy organized into physically effective form.**

This distinction is crucial for the maturity of the mathematical apparatus of the Theory of Entropicity (ToE). **It means that matter-energy is not to be postulated as an independent primitive standing outside entropy, but neither is it to be naïvely equated with [scalar] entropy in its simplest scalar form**. Rather, matter-energy emerges when **entropy becomes** (1) **locally constrained**, **directionally transported**, (2) **internally stored**, and (3) **dynamically resisted**. In this sense, (a) **mass represents stabilized entropic storage**; (b) **momentum represents directed entropic flow**; (c) **pressure represents entropic resistance to compression or rearrangement**; and (d) **stress represents the anisotropic distribution of entropic constraint within a physical system**. We recognize then that **the object that General Relativity calls the Stress-Energy Tensor ($T_{\mu\nu}$) can therefore be reinterpreted in the Theory of Entropicity (ToE) as the tensorial body of constrained entropy dynamics.**

This gives the right-hand side of Einstein's field equations a deeper meaning within the Theory of Entropicity (ToE). **The source of curvature is not merely "matter" in the conventional sense, but**

entropy that has become organized into matter-energy, pressure, momentum, and stress. **That is, according to the formulation of the Theory of Entropicity (ToE), the curvature side of Einstein's equation must express entropy geometrically, while the source side must express entropy materially and dynamically.** Thus, **the Einstein equation becomes, in the ToE limit, a correspondence between two modes of one underlying entropic reality**: **entropy as geometry and entropy as structured matter-energy.** Hence, **this transforms ToE from a general philosophical declaration that entropy causes gravity into a precise field-theoretic program:**

To derive the stress-energy tensor of Einstein's field equation itself from the entropic field, the Obidi Action, and the local laws of entropic accounting of the Theory of Entropicity (ToE).

End of Scholium of 6.3.1

6.1.3. Continuation of 6.1.3: Scholium on the RHS of the Einstein Field Equations and Its Implication for the Theory of Entropicity (ToE)

The coupling factor on the right-hand side of Einstein's equation is equally important:

$$\frac{8\pi G}{c^4}. \quad (6.2.26)$$

This factor determines how strongly matter-energy curves spacetime. In General Relativity, it is fixed by requiring that Einstein's equation reproduce Newtonian gravity in the weak-field limit. For ToE, however, the deeper ambition is to derive this coupling from entropic principles. If the Obidi Action is truly more fundamental than the Einstein–Hilbert action, then **(G), (c)**, and the combination $\boldsymbol{(8\pi G/c^4)}$ should not merely be imported as external constants. **They should arise as limiting parameters of the entropic field, the entropic speed limit, the entropic accounting structure, and the conversion between information constraint and curvature.**

This point connects directly with the **Entropic Accounting Principle (EAP)** of the Theory of Entropicity (ToE). In General Relativity, the Einstein tensor obeys the [celebrated] contracted **Bianchi identity,**

$$\nabla^\mu G_{\mu\nu} = 0. \quad (6.2.27)$$

Because the field equation equates $(G_{\mu\nu})$ with the stress-energy tensor, the source side must satisfy:

$$\nabla^\mu T_{\mu\nu} = 0. \quad (6.2.28)$$

This is the local conservation of energy and momentum. In the Theory of Entropicity (ToE), this conservation law becomes an **entropic accounting law (EAL).** We state the **entropic accounting law (EAL)** as follows:

The system cannot [no system can] create motion, mass, pressure, stress, or curvature without an entropic cost (EC).

If matter appears, entropy must be constrained. If motion appears, entropic capacity must be reallocated. If curvature appears, the field must account for the informational and energetic structure producing it. The conservation of ($T_{\mu\nu}$) therefore becomes the geometrized form of entropic bookkeeping.

This is one of the most powerful consequences of the right-hand side for ToE. It prevents the theory from becoming arbitrary. Entropy cannot simply be invoked as a universal explanation without mathematical discipline. The entropic source tensor must obey a local conservation or balance condition. It must couple consistently to curvature. It must reproduce the known stress-energy structure of matter, radiation, and pressure. It must reduce to Einstein's equation in the appropriate limit. And it must explain why local information-entropy reduction appears as matter while global entropic evolution appears as curvature and time direction.

In this refined interpretation, the Einstein equation becomes a correspondence relation inside the Theory of Entropicity (ToE).

Its more rigorous symbolic form is therefore

$$\frac{8\pi G}{c^4}\mathcal{T}^{(S)}_{\mu\nu}, (6.2.29)$$

where:

$$\mathcal{G}^{(S)}_{\mu\nu} \ (6.2.30)$$

denotes the entropic-geometric tensor obtained from the curvature structure generated by the entropic field, and

$$\mathcal{T}^{(S)}_{\mu\nu} \ (6.2.31)$$

denotes the entropic stress-energy tensor obtained from entropy as locally constrained, distributed, flowing, stored, and resisted. In the classical General Relativity limit, these two ToE objects must reduce respectively to the ordinary Einstein tensor and the ordinary stress-energy tensor:

$$\mathcal{G}\mu\nu^{(S)} \longrightarrow G\mu\nu, (6.2.32)$$

and

$$\mathcal{T}\mu\nu^{(S)} \longrightarrow T\mu\nu. (6.2.33)$$

Thus, the principal ToE declaration is not that entropy itself is directly equal to matter-energy, nor that scalar entropy is directly equal to curvature. The ToE declaration is that entropy, when processed through the Obidi Action and expressed in tensorial form, gives rise to two macroscopic structures: an entropic curvature tensor and an entropic source tensor. Their correspondence becomes Einstein's field equations in the appropriate classical limit.

The ToE correspondence may therefore be stated as follows:

$$\textbf{gravitational coupling} \times \textbf{entropic source tensor.}\ (\mathbf{6.2.34})$$

This formulation is superior because it preserves the tensorial structure, dimensional consistency, and physical meaning of Einstein's equation. It also clarifies **the task of the Theory of Entropicity (ToE)**:

To derive both the geometric tensor and the source tensor from entropy, rather than merely replacing spacetime curvature or matter-energy with the word "entropy."

In this sense, General Relativity appears as the classical limit of a deeper entropic-information dynamics [of the Theory of Entropicity (ToE)], where geometry and matter are not independent primitives but two tensorial manifestations of the same underlying entropic field.

This is the monistic force of ToE. General Relativity contains two great structures: geometry and matter-energy. The Theory of Entropicity (ToE) seeks to reduce both to a deeper entropic foundation. The left-hand side (LHS) of Einstein's Field Equations is the curvature expression of entropy.

The right-hand side (RHS) of Einstein's Field Equations is the material expression of entropy. The equation between them [that is, between the LHS and the RHS of the ToE formulation of

the Einstein Field Equations of General Relativity (GR)] is not merely a gravitational equation; it is a statement that the universe balances its geometric structure against its local entropic constraints.

The right-hand side (RHS) of Einstein's field equations therefore gives the Theory of Entropicity (ToE) its most demanding task. It is not enough to say that entropy curves spacetime (that is, the LHS of Einstein's field equations).

The Theory of Entropicity (ToE) must show (1) **how entropy becomes matter**, (2) **how information becomes mass**, (3) **how local entropic decrease becomes stress-energy**, (4) **how entropic pressure becomes gravitationally active**, and (5) **how the Obidi Action yields a conserved tensor that reduces to ($T_{\mu\nu}$).**

If ToE succeeds in doing this, then Einstein's equation will appear not as the final foundation of gravity, but as the classical macroscopic limit of a deeper entropic-information dynamics rooted in the foundations of the Theory of Entropicity (ToE) itself.

The deepest implication is therefore this: the right-hand side of Einstein's equation is where ToE must demonstrate and prove that matter is entropy in structured form. The left-hand side challenges ToE to explain curvature. The right-hand side challenges ToE to explain matter-energy.

The complete Theory of Entropicity (ToE) must explain both. Only then can ToE claim that gravity is not fundamentally the curvature of spacetime caused by matter, but the visible [physical] balance between two modes of entropy:

Entropy as information geometry and entropy as constrained material source.

6.2 What the Einstein–Hilbert Action Does Not Explain

The Einstein–Hilbert Action, for all its elegance, takes spacetime geometry as a primitive. It does not explain:

- Why the metric $g_{\mu\nu}$ exists or what determines its specific form;

- Why gravity couples to energy-momentum with strength $1/16\pi G$ [in the action];
- Why the cosmological constant has the value it does (the cosmological constant problem);
- How gravity is to be reconciled with quantum mechanics;
- The origin of the arrow of time in the gravitational sector.

The Theory of Entropicity (ToE) addresses all five of these open questions by revealing that the Einstein–Hilbert Action is not the deepest level of description — it is the effective action that emerges from the Obidi Action in a particular limit.

6.3 The Role of Action Principles in Physics

Action principles occupy a unique position in fundamental physics because they encode the deepest symmetries of a theory. From Noether's theorem, every continuous symmetry of the action implies a conservation law. **The action also determines the theory's propagators, vertex functions, and quantum corrections. To show that one action contains another as a special case is, therefore, to show that one theory is more fundamental than another in a precise, mathematical sense.**

In what follows, we establish this relationship: **the Einstein–Hilbert Action (6.1) emerges from the Obidi Action (3.1) in the near-equilibrium, weak-gradient, large-scale limit. The Obidi Action is the more fundamental of the two.**

7. The Obidi–Einstein Correspondence (OEC): From Information Geometry (IG) to Physical Gravity (PG)

In this section we explore the core mathematical result of this paper (Letter III of the ToE LRLS): a detailed derivation establishing the **formal correspondence between the Obidi Action and the Einstein–Hilbert Action, and demonstrating that Einstein's field equations emerge as a limiting case of the entropic field equations of the Theory of Entropicity (ToE).**

7.1 The Generalized Obidi Action with Geometric Sectors

To derive gravitational dynamics, we generalize the Obidi Action to include both the entropy field and the full geometric structure:

$$A_{\text{Obidi}}^{(\text{gen})}[S,\Phi;g,G] = A_{\text{ToE}}[S;g] + A_{\text{m}}[\Phi;g] + A_{\text{con}}[S,\Phi;g,G] \quad (7.1)$$

where:

- $A_{\text{ToE}}[S;g]$ is the initial restricted Obidi Action (3.1) for the entropy field;
- $A_{\text{m}}[\Phi;g]$ is the standard matter action, coupling matter fields Φ to the [entropic] metric;
- $A_{\text{con}}[S,\Phi;g,G]$ is a constraint action enforcing entropic consistency between the entropy-weighted metric sector and the matter-induced metric sector.

The constraint action is:

$$A_{\text{con}}[S,\Phi;g,G] = \int d^4\, x, \sqrt{-g}, G^{\mu\nu}\Big(G_{\mu\nu}^{(S)}[S;g] \; - m_{\mu\nu}[\Phi;g]\Big) \quad (7.2)$$

where $G_{\mu\nu}^{(S)}[S;g]$ is the entropy-weighted metric defined in (4.3), $m_{\mu\nu}[\Phi;g]$ is the matter-induced metric (a functional of matter fields Φ and the background metric), and Gμν is a tensorial Lagrange multiplier. Variation of (7.2) with respect to $G^{\mu\nu}$ enforces:

$$G_{\mu\nu}^{(S)}[S;g] \; = \; m_{\mu\nu}[\Phi;g] \quad \textit{(7.3)}$$

This constraint ensures entropic consistency between the geometric and matter sectors — it is the fundamental reason why the gravitational and matter sectors are coupled in the emergent effective theory.

7.2 Metric Variation and the Dressed Einstein Equations

Variation of the generalized Obidi action (7.1) with respect to the metric $g_{\mu\nu}$, using **the Palatini identity** and the definitions of the stress-energy tensors from Sections 3.4 and 7.1, yields the dressed Einstein equations:

$$G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu} = 8\pi G_{\text{eff}} \left[T_{\mu\nu}^{(\text{m})} + T_{\mu\nu}^{(S)} + T_{\mu\nu}^{(\mathcal{G})} \right] \quad (7.4)$$

where:

- $G_{\mu\nu}$ is the Einstein tensor;
- $\Lambda_{\text{ent}} = \langle(\nabla S)^2\rangle$ is the entropic cosmological term, equal to the coarse-grained variance of entropy gradients over cosmological scales;
- G_{eff} is the effective gravitational coupling, determined by the entropic coupling constant η;
- $T_{\mu\nu}^{(S)}$ is the entropic stress-energy tensor (3.6);
- $T_{\mu\nu}^{(\mathcal{G})}$ is the G-field stress-energy tensor arising from the constraint sector (7.2).

Equation (7.4) is the ToE generalization of Einstein's field equations of General Relativity (GR). It contains three distinct stress-energy contributions (matter, entropy field, and constraint sector) and an emergent cosmological term. Crucially, Λ_{ent} is not a free parameter — it is dynamically determined by the entropic field, resolving the cosmological constant problem in principle.

7.2.1 Scholium on the Palatini Identity

The Palatini identity is used in the variation of the generalized Obidi action to control the metric variation of the curvature terms. Since the Ricci tensor depends on second derivatives of the metric through the Levi-Civita connection, its direct variation would otherwise produce derivative terms in $\delta g_{\mu\nu}$. The Palatini identity rewrites the variation of the Ricci tensor as a covariant divergence of the variation of the connection. Under integration by parts, this divergence contributes only a boundary term, which is discarded under suitable boundary conditions or cancelled by an appropriate boundary action. The remaining bulk variation of the curvature sector is therefore proportional to the Einstein tensor $G_{\mu\nu}$. When the entropic vacuum contribution is included, this produces the geometric side $G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu}$. The remaining variations of the matter, entropic, and information-geometric sectors define the corresponding stress-energy tensors $T_{\mu\nu}^{(m)}$, $T_{\mu\nu}^{(S)}$, and $T_{\mu\nu}^{(\mathcal{G})}$. Setting the total variation of the generalized Obidi action to zero therefore yields the dressed Einstein equations, in which curvature is sourced not [only] by ordinary matter but [also] by the entropic field and the information-geometric constraint structure. Natural units have been imposed throughout.

7.2.1 Scholium on the Mathematical Apparatus of the Palatini Identity

The **Palatini identity** is the standard identity used to vary curvature with respect to the metric. It explains why varying a curvature action produces the Einstein tensor rather than an uncontrolled expression involving derivatives of the metric variation.

The identity is:

$$\delta R_{\mu\nu} = \nabla_\lambda \delta\Gamma^\lambda_{\mu\nu} - \nabla_\nu \delta\Gamma^\lambda_{\mu\lambda}. \,(7.4.1)$$

Here $R_{\mu\nu}$is the Ricci tensor, $\Gamma^\lambda_{\mu\nu}$is the Christoffel connection, and $\nabla_\mu$is the covariant derivative compatible with the metric.

In simple terms, the Palatini identity posits that:

The variation of the Ricci tensor can be written as a covariant divergence of the variation of the connection.

That is important because divergence terms become **boundary terms** inside an action integral. After integration by parts, and assuming suitable boundary conditions, those boundary terms do not contribute to the bulk field equations. This is what allows the metric variation of the Einstein-Hilbert curvature term to produce the Einstein tensor.

The key result is:

$$\delta\left(\sqrt{-g}R\right) = \sqrt{-g}\, G_{\mu\nu}\delta g^{\mu\nu} + \text{boundary term}. \,(7.4.2)$$

After the boundary term is discarded or cancelled by an appropriate boundary action, the surviving bulk term is:

$$\sqrt{-g}\, G_{\mu\nu}\delta g^{\mu\nu}. \,(7.4.3)$$

This is the mathematical reason why variation of the curvature action gives:

$$G_{\mu\nu}. \,(7.4.4)$$

So, in our case in the earlier section, the Palatini identity is being used to justify the step from the variation of the curvature-containing part of the generalized Obidi action to the geometric left-hand side of the dressed Einstein equations.

Recollect that our final equation is:

$$G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu} = 8\pi G_{\text{eff}} \left[T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(S)} + T_{\mu\nu}^{(\mathcal{G})} \right]. \quad (7.4.5)$$

The role of the Palatini identity is mainly on the **left-hand side**. It explains how the variation of the curvature term in the generalized Obidi action produces $G_{\mu\nu}$. The entropic cosmological term $\Lambda_{\text{ent}} g_{\mu\nu}$ comes from varying an entropic vacuum-density term such as:

$$\int d^4x \sqrt{-g}\, \Lambda_{\text{ent}}. \quad (7.4.6)$$

The right-hand side comes from the metric variation of the entropic source sectors of the action. In standard form, each stress-energy tensor is defined by:

$$T_{\mu\nu}^{(i)} = -\frac{2}{\sqrt{-g}} \frac{\delta A_i}{\delta g^{\mu\nu}}, \quad (7.4.7)$$

where i may represent ordinary matter, the entropic field, or the information-geometric/coupling sector.

Thus:

$$T_{\mu\nu}^{(m)} \quad (7.4.8)$$

comes from varying the ordinary matter action (entropic in its strict formulation),

$$T_{\mu\nu}^{(S)} \quad (7.4.9)$$

comes from varying the entropic-field sector,

and

$$T_{\mu\nu}^{(\mathcal{G})} \quad (7.4.10)$$

comes from varying the [entropic] information-geometric or [entropic] constraint sector.

Hence, the logic is:

$$\delta A_{\text{Obidi}}^{(\text{gen})} = 0 \quad (7.4.11)$$

implies:

$$\delta A_{\text{gravity}} + \delta A_{\text{entropic}} + \delta A_{\text{matter}} + \delta A_{\text{constraint}} = 0. \quad (7.4.12)$$

The Palatini identity handles the variation of the curvature piece:

$$\delta A_{\text{gravity}} \longrightarrow G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu}. \, (7.4.13)$$

The stress-energy definitions handle the source pieces:

$$\delta A_{\text{source}} \longrightarrow T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(S)} + T_{\mu\nu}^{(\mathcal{G})}. \, (7.4.14)$$

Therefore, the field equation becomes a dressed Einstein equation in which ordinary matter is not the only source. The source is enlarged to include matter, the entropic field, and the information-geometric constraint sector. **In the full Theory of Entropicity (ToE) program, the task remains to recast all the source terms in terms of the Entropic Field [of entropy] itself.**

7.3 The Near-Equilibrium Limit: Recovery of Einstein's Equations

We now demonstrate the recovery of Einstein's standard field equations (6.2) from (7.4) in the appropriate limit. Consider fluctuations about a local equilibrium configuration Seq(x) satisfying:

$$\frac{dV}{dS}\bigg| S_{\text{eq}} = J(x) \quad \text{and} \quad \nabla_\mu \nabla^\mu S_{\text{eq}} = 0. \, (7.5)$$

Or

$$\frac{dV}{dS}\bigg| S_{\text{eq}} = J(x) \quad \text{and} \quad \Box S_{\text{eq}} = 0. \, (7.5.1)$$

Write S(x) = S_{eq}(x) + δS(x) with δS small. Taylor-expanding the potential:

$$V(S) \approx V(S_{\text{eq}}) + \tfrac{1}{2} M^2 S(x) (\delta S)^2 + O(\delta S^3) \quad \textit{(7.6)}$$

where $M^2S(x) = d^2V/dS^2|S_{\text{eq}}$ is the local entropic "mass squared" (the local curvature of the entropic potential).

In this limit, the entropic kinetic term in A_{ToE} becomes proportional to $(\nabla\delta S)^2$, the entropic potential contributes a constant $V(S_{\text{eq}}) \rightarrow \Lambda_{\text{ent}}$ (the cosmological term), and **the quadratic term reproduces Bianconi's relative-entropy functional.** Moreover, the entropic stress-energy tensor reduces to:

$$T_{\mu\nu}^{(S)} \rightarrow 0 \quad \text{[when } \nabla S \rightarrow 0 \text{ and } V'(S) \rightarrow \text{constant]} \quad \textit{(7.7)}$$

In this same limit, the constraint equation (7.3) is satisfied automatically, so $T_{\mu\nu}^{(\mathcal{G})}$ effectively vanishes or is absorbed into a renormalization of G_{eff}. Equation (7.4) then reduces to:

$$G_{\mu\nu} + V\big(S_{\mathrm{eq}}\big)g_{\mu\nu} = 8\pi G_{\mathrm{eff}}T_{\mu\nu}^{(\mathrm{m})}$$

which is precisely Einstein's equation (6.2) with:

$$\Lambda = \mathrm{V}(S_{\mathrm{eq}}) = \Lambda_{\mathrm{ent}} \quad \text{and} \quad \mathrm{G} = G_{\mathrm{eff}} \quad \textit{(7.9)}$$

This establishes the formal result: **Einstein's field equations are the near-equilibrium, weak-gradient, classical limit of the ToE entropic field equations derived from the Obidi Action.**

7.4 The Obidi Action as the Parent Action

We can now state the structural correspondence precisely. Define the following limits:

L_1: ∇S → 0, V'(S) → const [classical, static entropy] *(7.10)*

L_2: δS small [near-equilibrium fluctuations] *(7.11)*

L_3: α → 0, (α, q) → (1, 1) [Boltzmann–Gibbs / Fisher–Shannon sector] *(7.12)*

Then:

$$A_{\mathrm{ToE}} \overset{L_1}{\to} S_{\mathrm{EH}} + \Lambda_{\mathrm{ent}} \int d^4\, x, \sqrt{-g} \text{ (7.13)}$$

$$A_{(ToE)} \xrightarrow{(L_2)} I_e f f^{((B))} + A_E H^{((GR))} [Bianconi + GR] \text{ (7.14)}$$

This means that: under the second limiting procedure L_2, the ToE action reduces to an effective Bianconi-type (B) information action plus the Einstein–Hilbert gravitational action of General Relativity (GR).

$$A_{(ToE)} \xrightarrow{(L_1+L_2+L_3)} S_{EH} \; [pure\ General\ Relativity] \text{ (7.15)}$$

These results establish the hierarchy:

Einstein GR ⊂ Bianconi Entropic Gravity ⊂ Obidi's Theory of Entropicity (ToE)

Each inclusion corresponds to a successive relaxation of assumptions: from static entropy and Boltzmann–Gibbs statistics (GR), to informational but time-symmetric entropy (Bianconi), to fully dynamic, irreversible, and universal entropic field theory (ToE).

7.5 Formal Table of Correspondences

The structural parallels between the Einstein–Hilbert Action and the Obidi Action are summarized in Table 2 below:

Einstein–Hilbert Action (GR)	Obidi Action (ToE)
Fundamental field: metric $g_{\mu\nu}$	Fundamental field: entropy S(x)
Action: SEH = (1/16πG) ∫ √(−g) R d⁴x	Action: AToE = ∫ √(−g) [χ(∇S)² − V(S) + J·S] d⁴x
Geometric degree of freedom: R (Ricci scalar)	Entropic degree of freedom: $(\nabla S)^2$ (entropy gradient squared)
Field equations: $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T^{(m)}{}_{\mu\nu}$	Entropic equations: $G_{\mu\nu} + \Lambda_{ent}\, g_{\mu\nu} = 8\pi G_{eff}\, [T^{(m)}{}_{\mu\nu} + T^{(S)}{}_{\mu\nu} + T^{(G)}{}_{\mu\nu}]$
Levi–Civita connection: ∇LC	Amari–Čencov α-connections → ∇(α) → ∇LC (emergent)
Cosmological constant Λ: free parameter	Entropic Λent = ⟨(∇S)²⟩: dynamically determined
Coupling G: fundamental constant	Geff: derived from entropic coupling η
Spacetime metric: fundamental	Spacetime metric: emergent from S(x)
Matter action SM[Φ, g]: external input	Matter action: emergent from entropic projections of S(x)
GR: no arrow of time	ToE: arrow of time from Vuli–Ndlela irreversibility functional
No information-theoretic foundation	Fisher–Rao + Fubini–Study + α-connections: complete info-geometric foundation

Table 2: Structural correspondence between the Einstein–Hilbert Action of GR and the Obidi Action of ToE.

8. Information Geometry as the Origin of Gravity: The Complete Pipeline of the Theory of Entropicity (ToE)

Having established the formal correspondence, we can now articulate the complete logical and mathematical pipeline by which information geometry gives rise to physical gravity. This pipeline is the central conceptual contribution of ToE, and it is what distinguishes it from all previous entropic approaches to gravity.

8.1 The Seven-Stage Pipeline

The transformation from raw entropic field to emergent physical spacetime proceeds through seven logically distinct and mathematically precise stages:

1. **Ontic Entropy Field:** The primitive object is the scalar entropy field S(x) on a pre-metric manifold M. This is the fundamental degree of freedom of the universe. It has no background spacetime — it generates spacetime.
2. **Informational Lift:** The entropy field S(x) induces a family of probability distributions p(·|θ; S) over physical configurations. This lifts S(x) into the domain of information geometry, providing access to the Fisher–Rao metric (classical) and the Fubini–Study metric (quantum).
3. **HMAS Construction**: The Hybrid Metric-Affine Space is assembled from the Fisher–Rao and Fubini–Study metrics, providing a unified metric structure that contains both classical and quantum sectors. This is the geometric arena of the full ToE theory.
4. **Rényi–Tsallis Deformation:** Non-additive entropies (Rényi, Tsallis) introduce the deformation parameter q, which curves the statistical manifold — encoding long-range correlations, non-extensivity, and multi-scale behavior. This curvature becomes the seed of physical curvature.
5. **α-Connection Encoding:** The Amari–Čencov α-connections encode the Rényi–Tsallis deformation geometrically, via the constitutive constraint $\alpha = 2(1-q)$. They equip the information manifold with affine structure — directional transport, curvature, and the arrow of time.
6. **Obidi Action Dynamics:** The Obidi Action embeds the HMAS structure into a variational principle, coupling the entropy field to the geometric degrees of freedom. Extremization

yields the Master Entropic Equation, entropic geodesics, and entropy potential equation. The Obidi Action is the selection principle that determines which α-connection and which metric describe the physical world.

7. **Levi–Civita Projection and GR:** In the classical, large-scale limit, the α-connection reduces to the Levi–Civita connection of a Riemannian spacetime. The entropic field equations reduce to Einstein's field equations. Physical spacetime, with its familiar metric and gravitational dynamics, emerges as the effective large-scale description of the entropic manifold.

This pipeline can be summarized in the following chain, which constitutes the fundamental theorem of the Theory of Entropicity (ToE):

S(x) → p(·|θ;S) → {g(FR), g(FS)} → [Tsallis/Rényi, q] → [Amari–Čencov, α = 2(1−q)] →

$$A_{(ToE)} \rightarrow G_{(\mu\nu)}, T_{(\mu\nu)}^{((S))}, \Lambda_{(ent)} \rightarrow S_{EH} + corrections$$

8.2 Why the Obidi Action is Indispensable

A common misunderstanding is that the Amari–Čencov α-connections alone could transform the Fisher–Rao/Fubini–Study information manifold into physical spacetime. This is not the case. The α-connections provide the kinematics of the information manifold — they describe how information flows and how the manifold is curved. But kinematics is not dynamics. The α-connections do not, by themselves:

- Select which α corresponds to the physical connection;
- Provide field equations determining how the entropy field and metric co-evolve;
- Yield conservation laws (which require an underlying action symmetry);
- Enforce the metric-compatibility and torsion-free conditions that define the Levi–Civita connection.

The Obidi Action accomplishes all four of these. It is the selection principle, the generator of dynamics, the source of Noether conservation laws, and the enforcer of the Levi–Civita limit. The analogy is precise: just as the Einstein–Hilbert Action does for the metric what the Amari–Čencov connections cannot do for statistical manifolds, the Obidi Action does for the entropy field what no purely kinematic information-geometric construction can achieve.

In this sense, the Obidi Action performs for entropy what the Einstein–Hilbert Action performs for curvature — and goes further, because it reveals that the Einstein–Hilbert Action is itself a consequence of the Obidi Action.

9. Derivation of Einstein Geometry and the Entropic Source Tensor of the Einstein Field Equations of General Relativity from the Theory of Entropicity (ToE)

9.1 Introduction to the Mathematical Framework of the Theory of Entropicity (ToE)

The mathematical task announced in section 6 above in this Letter III is very precise: ToE must derive not only the geometric side of Einstein's field equations, but also the source side, so that curvature and matter-energy appear as two tensorial projections of one deeper entropic-information substrate. This Letter III already formulates that demand explicitly: the theory is not complete unless it yields both an entropic curvature tensor and an entropic source tensor whose infrared classical limit is Einstein's equation. In this section of our work, we now wish to proceed to sharpen that program into a single derivational machinery whose end-point is:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(\mathrm{m})}. \quad (9.1.1)$$

In what follows, uppercase Latin indices $A, B, \ldots$ refer to the information manifold, while Greek indices $\mu, \nu, \ldots$ refer to the emergent four-dimensional spacetime. The metric signature is $(-,+,+,+)$. To avoid sign ambiguities in the scalar sector, the kinetic invariant of the entropy field is defined by

$$X := -\frac{1}{2}\nabla_\mu S \nabla^\mu S. \quad (9.1.2)$$

For timelike entropic flow, $X > 0$. This is the standard covariant normalization used in scalar-fluid correspondences and makes the stress-tensor formulas unambiguous.

9.2 Information-Geometric Starting Point

The information-geometric input is a statistical manifold $\mathcal{M}_I$ whose points are probability distributions $p(\omega \mid \Theta, S)$, with coordinates Θ^A, and where the entropy field S acts as the ontological control field selecting or weighting admissible probability sectors. The canonical metric on $\mathcal{M}_I$ is the Fisher–Rao metric,

$$I_{AB}(\Theta, S) = \int_\Omega p\,(\omega \mid \Theta, S)\ \partial_A \ln p(\omega \mid \Theta, S)\ \partial_B \ln p(\omega \mid \Theta, S)\, d\mu(\omega). \quad (9.2.1)$$

Its associated cubic information tensor is the Amari–Čencov tensor,

$$C_{ABC} = \int_{\Omega} p \; \partial_A \ln p \; \partial_B \ln p \; \partial_C \ln p \; d\mu, (9.2.2)$$

and the one-parameter family of α-connections may be written schematically as

$$\Gamma^{(\alpha)}_{ABC} = \Gamma^{(0)}_{ABC} - \frac{\alpha}{2} C_{ABC}, (9.2.3)$$

with $\Gamma^{(0)}$ the **Levi–Civita connection** of I_{AB}. In classical information geometry, the Fisher metric is singled out by uniqueness theorems of the **Čencov type**: up to scale, it is the **unique monotone Riemannian information** metric compatible with **sufficient statistics** or **Markov morphisms**. This is exactly why it is powerful, but it is also why it is insufficient by itself for relativistic spacetime: it is positive-definite and therefore cannot by itself carry a Lorentzian causal cone.

This is the precise point at which ToE departs from ordinary information geometry. The geometry of distinguishability must be converted into a geometry of causality. In the current Letter III, that move is attributed to the **Obidi transformation**; mathematically it is a **disformal, rank-one**, entropy-gradient deformation of the information metric. **Bekenstein's** work on the relation between physical and gravitational geometry and later disformal-transformation studies show that such gradient-driven metric deformations are a mathematically legitimate operation in gravitational theory; **ToE's novelty is to identify the entropy gradient itself as the preferred direction and to make that deformation the ontological origin of time-orientation.**

Accordingly, we define the normalized entropic direction on $\mathcal{M}_I$ by

$$u_A = \frac{\nabla_A S}{\sqrt{I^{BC} \nabla_B S \nabla_C S}}, \qquad I^{AB} u_A u_B = 1. (9.2.4)$$

The minimal **Lorentzianized information metric (LIM)** is then

$$\tilde{G}_{AB} = \Omega^2(\Theta, S) \, (I_{AB} - 2 u_A u_B) + \varepsilon \, Q_{AB}, (9.2.5)$$

where Q_{AB} represents the optional **quantum-information correction sector** and may be taken, in the simplest case, as zero. In an I_{AB}-orthonormal frame adapted to u_A, the tensor $I_{AB} - 2u_A u_B$ becomes $\mathrm{diag}(-1, 1, \dots, 1)$. Hence the entropy gradient flips exactly one eigenvalue and produces a Lorentzian signature. If εQ_{AB} is a perturbatively small correction, the signature is stable. **This is the exact mathematical reason ToE can pass from Fisher distinguishability to relativistic causal geometry.**

9.3 Lorentzian Emergence and the Effective Geometric Action

The **emergent four-dimensional spacetime** M is taken to be a **macroscopic image** of the information manifold under **an embedding or emergence map**

$$X: M \to \mathcal{M}_I, \qquad x^\mu \mapsto X^A(x). (9.3.1)$$

The physical spacetime metric is the pullback of the **Lorentzianized information metric (LIM)**:

$$g_{\mu\nu}(x) = \partial_\mu X^A \, \partial_\nu X^B \, \tilde{G}_{AB}\big(X(x)\big). (9.3.2)$$

Thus, the statement "**entropy generates spacetime**" becomes mathematically precise: **the metric that defines intervals, geodesics, and causal cones is induced from information geometry only after the entropic sign-flip.** This formal step is the **tensorial backbone of ToE's** claim that **geometry is not primitive but emergent from the entropy field.**

Now place on $\mathcal{M}_I$ the **parent information-gravity action (PIGA)**

$$A_{\mathrm{IG}} = \frac{1}{2\kappa_I} \int_{\mathcal{M}_I} d^N \Theta \, \sqrt{|\tilde{G}|} \, \big(\mathcal{R}[\tilde{G}] - 2\Lambda_I\big), (9.3.3)$$

where $\mathcal{R}[\tilde{G}]$ is the **scalar curvature** of the **Lorentzianized information metric (LIM)**, Λ_I is the **parent information-vacuum (PIV)** term, and κ_I is the fundamental **entropic-information coupling** (**EIC**). If $\mathcal{M}_I$ is **foliated** by **four-dimensional macroscopic leaves** $X(M)$ with transverse fibers $\mathcal{F}_x$, then the **Gauss–Codazzi** decomposition implies that the **higher-dimensional scalar curvature** splits into an **intrinsic four-dimensional Ricci scalar** plus **transverse-curvature**, **extrinsic-curvature**, and **boundary pieces**:

$$\mathcal{R}[\tilde{G}] = R[g] + \mathcal{U}_\perp + \nabla_A V^A. (9.3.4)$$

Here $\boldsymbol{\mathcal{U}_\perp}$ is shorthand for the complete scalar built from **normal-bundle curvature** and **extrinsic curvature**, and $\boldsymbol{\nabla_A V^A}$ is a **total divergence**. This **decomposition is the geometric lever that turns information curvature into spacetime curvature. Analogous induced-gravity reductions are standard in embedding and brane constructions.**

After integration over the **transverse fibers** and omission of the **total-divergence contribution** under standard boundary assumptions, the **parent action** reduces to an **effective four-dimensional geometric action** of the form:

$$A^{(4)}_{\mathrm{geom}} = \int_M d^4 x \, \sqrt{-g} \, \left[\frac{1}{16\pi G_{\mathrm{eff}}(x)} \, R[g] - \Lambda_{\mathrm{ent}}(x) + \mathcal{L}^{\mathrm{geo}}_{\mathrm{corr}}\right]. (9.3.5)$$

It is convenient to define this reduction by two scalar functionals Z_R and Z_Λ:

$$\frac{1}{16\pi G_{\text{eff}}(x)} := \frac{Z_R(x)}{2\kappa_I}, \qquad \Lambda_{\text{ent}}(x) := \frac{Z_\Lambda(x)}{Z_R(x)}. \ (9.3.6)$$

The quantity Z_R is the **fiber-integrated coefficient** multiplying the four-dimensional **Ricci scalar**, while Z_Λ collects the **fiber-averaged information-vacuum term**, the **equilibrium entropy potential**, and the **transverse/extrinsic scalar contributions**. In the strict infrared **Einstein sector**, Z_R becomes **effectively constant** and Λ_{ent} becomes the **macroscopic cosmological term**. This is the **precise sense in which the Einstein–Hilbert functional appears as the universal two-derivative truncation of information gravity in the Theory of Entropicity (ToE).**

9.4 Derivation of the Geometric Curvature Tensor Left-Hand Side (LHS) of Einstein's Field Equations of General Relativity (GR) from the Theory of Entropicity (ToE)

At the four-dimensional level, the pure geometric part of the effective action is therefore

$$A^{(4)}_{\text{geom}} = \frac{1}{16\pi G_{\text{eff}}} \int_M d^4 x \sqrt{-g}\, (R - 2\Lambda_{\text{ent}}) + \int_M d^4 x \sqrt{-g}\, \mathcal{L}^{\text{geo}}_{\text{corr}}, \ (9.4.1)$$

where, for the Einstein limit, G_{eff} may be treated as constant over the macroscopic domain of interest. Varying with respect to $g^{\mu\nu}$ and using the **Palatini identity** gives:

$$\delta \int d^4x \sqrt{-g}\, R = \int d^4x \sqrt{-g}\, G_{\mu\nu}\, \delta g^{\mu\nu}, \ (9.4.2)$$

up to the usual boundary term. Likewise,

$$\delta \int d^4x \sqrt{-g}\, (-2\Lambda_{\text{ent}}) = \int d^4x \sqrt{-g}\, \Lambda_{\text{ent}}\, g_{\mu\nu}\, \delta g^{\mu\nu}, \ (9.4.3)$$

provided Λ_{ent} is treated as a slowly varying or constant background at the level of the **macroscopic variation.** Any **residual metric** dependence of G_{eff}, Λ_{ent}, or the higher-order sector is absorbed into a **correction tensor:**

$$H^{\text{corr}}_{\mu\nu} := -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \left(\sqrt{-g}\, \mathcal{L}^{\text{geo}}_{\text{corr}}\right). \ (9.4.4)$$

Hence the left-hand side of Einstein's Field Equations (of GR) generated by ToE is obtained as:

$$\mathcal{E}^{\text{geom}}_{\mu\nu} = G_{\mu\nu} + \Lambda_{\text{ent}}\, g_{\mu\nu} + H^{\text{corr}}_{\mu\nu}. \ (9.4.5)$$

This is already the **Einstein tensor structure**, now understood as the **macroscopic curvature image** of the **Lorentzianized information manifold** (**LIM**) rather than as a fundamental postulate. In the **Einstein limit**, $H_{\mu\nu}^{\text{corr}} \to 0$, leaving the standard left-hand side of GR.

The **contracted Bianchi identity** then guarantees

$$\nabla^{\mu} G_{\mu\nu} = 0, (9.4.6)$$

Hence, the full ToE equation must satisfy a **generalized balance law**. If G_{eff} and Λ_{ent} are constant and the correction sector is negligible, one recovers the **standard conservation law**

$$\nabla^{\mu} T_{\mu\nu} = 0. (9.4.7)$$

This is not an optional addition; it is the consistency condition that forces the right-hand side also to be tensorial and conserved. It is exactly the point emphasized in section 6 of this Letter III: the source side must be derived with the same rigor as the curvature side.

9.5 Derivation of the Entropic Stress-Energy Tensor Right-Hand Side (RHS) of Einstein's Field Equations of General Relativity (GR) from the Theory of Entropicity (ToE)

The **decisive step of the Theory of Entropicity (ToE)** is to recognize that a **single scalar entropy field (SSEF)** is not by itself the **whole material side of Einstein's equation**. A **single scalar** can generate a **vacuum sector** and, when its gradient is timelike, it can also generate an **effective perfect fluid**; but the full **Einstein source tensor** (**EST**) also contains **momentum flux**, **pressure**, **shear**, and **radiation**. Therefore, the correct ToE completion of the right-hand side is not "**entropy equals $\boldsymbol{T_{\mu\nu}}$**" in a naïve scalar sense. **It is that the entropy field, together with its localized currents, fluctuations, and propagating information distribution, generates a rank-two tensor by metric variation and by coarse-grained second moments. This is the exact completion that this Letter III of the Theory of Entropicity (ToE) calls for.**

Now, with the same logic given in the foregoing sections, we write the **effective source action** as

$$A_{\text{src}} = \int_M d^4 x \sqrt{-g} \left[F(X, S) - \frac{\lambda_C}{2} C + \mathcal{L}_{\text{flux}}[f_{\text{ent}}, g] + \mathcal{L}_G \right], (9.5.1)$$

with

$$X = -\frac{1}{2} \nabla_{\mu} S \nabla^{\mu} S, \qquad C := g^{\mu\nu} C_{\mu\nu}. (9.5.2)$$

Here $F(X,S)$ is the **coherent entropic-storage sector** (**CESS**), $C_{\mu\nu}$ is a **symmetric information-covariance tensor** (**SICT**) describing **local statistical fluctuations** of **entropic currents**, $f_{\text{ent}}(x,\pi)$ is the coarse-grained distribution of **propagating information carriers** on the **cotangent** or **momentum bundle**, and $\mathcal{L}_G$ denotes the ToE constraint sector already discussed earlier. The Theory of Entropicity (ToE) **total entropic source tensor** (**TEST**) is then defined, exactly as in any covariant field theory, by:

$$T_{\mu\nu}^{\text{ent}} = -\frac{2}{\sqrt{-g}}\frac{\delta A_{\text{src}}}{\delta g^{\mu\nu}} = T_{\mu\nu}^{(S)} + T_{\mu\nu}^{(C)} + T_{\mu\nu}^{(\text{flux})} + T_{\mu\nu}^{(G)}. (9.5.3)$$

The **right-hand side of Einstein's equation** is thus not inserted but generated.

The **coherent scalar sector** (**CSS**) follows directly from varying $A_S = \int \sqrt{-g}\, F(X,S)\, d^4x$. Since

$$\delta X = -\frac{1}{2}\nabla_\mu S \nabla_\nu S\, \delta g^{\mu\nu}, (9.5.4)$$

one finds

$$\delta A_S = -\frac{1}{2}\int d^4x \sqrt{-g}\, \left(F_X \nabla_\mu S \nabla_\nu S + F g_{\mu\nu}\right)\delta g^{\mu\nu}, (9.5.5)$$

and therefore

$$T_{\mu\nu}^{(S)} = F_X \nabla_\mu S \nabla_\nu S + F g_{\mu\nu}. (9.5.6)$$

If $\nabla_\mu S$ is timelike, we can define

$$u_\mu = \frac{\nabla_\mu S}{\sqrt{2X}}, \qquad u^\mu u_\mu = -1. (9.5.7)$$

Then

$$T_{\mu\nu}^{(S)} = (\rho_S + p_S) u_\mu u_\nu + p_S g_{\mu\nu}, (9.5.8)$$

with

$$p_S = F, \qquad \rho_S = 2XF_X - F. (9.5.9)$$

Hence the entropy field by itself already yields a perfect-fluid sector: (1) **energy density from stored entropic organization**, and (2) **isotropic pressure from local entropic response**. This is mathematically the same **scalar-fluid correspondence (SFC)** well known in relativistic field theory (**RFT**), now interpreted ontologically within the Theory of Entropicity (ToE).

The fluctuation sector is the minimal tensorial completion needed to represent internal stress and pressure beyond the coherent scalar piece. Let

$$C_{\mu\nu} = \langle \delta J_\mu \, \delta J_\nu \rangle_{\rm cg}, \qquad \delta J_\mu := J_\mu - \langle J_\mu \rangle_{\rm cg}, \, (9.5.10)$$

be the coarse-grained covariance of entropic currents. Varying the minimal quadratic term $-\frac{\lambda_C}{2}\int \sqrt{-g}\, C\, d^4x$ gives

$$T^{(C)}_{\mu\nu} = \lambda_C \left(C_{\mu\nu} - \frac{1}{2} C\, g_{\mu\nu} \right). \, (9.5.11)$$

This sector is the natural home of **internal pressure**, **elastic response**, **anisotropic stress**, and the **local resistance of organized information to rearrangement**. In the physical language of the Theory of Entropicity (ToE): **mass is stabilized entropic storage**, while **pressure** and **shear** are the **spatial covariance structure of that storage under coarse-graining.**

The **flux sector** is the **tensor of propagating informational transport**. Let $f_{\rm ent}(x,\pi)$ be a **covariant distribution of entropic carriers** on the local momentum space $\mathcal{P}_x$. Its **second moment** defines

$$T^{(\rm flux)}_{\mu\nu}(x) = \int_{\mathcal{P}_x} d\,\Pi\, \pi_\mu \pi_\nu\, f_{\rm ent}(x,\pi). \, (9.5.12)$$

This is the most direct mathematical realization of the central idea of the Theory of Entropicity (ToE): radiation, **directed flow**, and **momentum transport are localized and propagating information distributions, and their macroscopic signature is a rank-two tensor.**

When the support of $f_{\rm ent}$ is concentrated on timelike momenta, this sector behaves as matter flow; when it is concentrated on null momenta, one obtains a radiation tensor of the form:
$T^{(\rm rad)}_{\mu\nu} = \Phi\, k_\mu k_\nu, \qquad k^\mu k_\mu = 0. \, (9.5.13)$
Thus, mass-energy and radiation are not separate substances in Theory of Entropicity (ToE). They are different coarse-grained support conditions on the same entropic-information distribution.
The ToE **total entropic source tensor (TEST)** can now be decomposed in the standard imperfect-fluid way:
$T^{\rm ent}_{\mu\nu} = \rho_{\rm ent} u_\mu u_\nu + p_{\rm ent} h_{\mu\nu} + 2u_{(\mu} q_{\nu)} + \pi_{\mu\nu}, \qquad h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu, \, (9.5.14)$

where:
$$\rho_{\text{ent}} = u^{\mu}u^{\nu}T_{\mu\nu}^{\text{ent}}, (9.5.15)$$
$$q_{\mu} = -h_{\mu}{}^{\alpha}u^{\beta}T_{\alpha\beta}^{\text{ent}},$$
$$p_{\text{ent}} = \frac{1}{3}h^{\mu\nu}T_{\mu\nu}^{\text{ent}},$$
$$\pi_{\mu\nu} = \left(h_{(\mu}{}^{\alpha}h_{\nu)}{}^{\beta} - \frac{1}{3}h_{\mu\nu}h^{\alpha\beta}\right)T_{\alpha\beta}^{\text{ent}}.$$
These are precisely the tensorial slots that GR interprets as energy density, momentum density, isotropic pressure, and anisotropic stress. The **Theory of Entropicity (ToE)** claim is therefore mathematically sharper and much more logically motivated than a slogan: **Einstein's $T_{\mu\nu}$ is recovered as the coarse-grained tensor of constrained, fluctuating, and propagating entropy on the emergent spacetime from the Entropic Field.**

9.6 Einstein Limit of General Relativity and Its Physical Formulation from the Tensorial Sectors of the Theory of Entropicity (ToE)

Thus, from all of the above in the preceding sections, combining the **geometric** and **source sectors** gives the **four-dimensional Theory of Entropicity (ToE)** field equations [the **Obidi Field Equations (OFE)**]:

$$G_{\mu\nu} + \Lambda_{\text{ent}}g_{\mu\nu} + H_{\mu\nu}^{\text{corr}} = 8\pi G_{\text{eff}}\left(T_{\mu\nu}^{(S)} + T_{\mu\nu}^{(C)} + T_{\mu\nu}^{(\text{flux})} + T_{\mu\nu}^{(G)}\right). (9.6.1)$$

This is the mathematically complete ToE version of the dressed Einstein equation. **The left-hand side is generated from Lorentzianized information curvature.**

The right-hand side is generated from the coherent field, the covariance tensor of entropic fluctuations, the transport tensor of propagating information, and the constraint sector enforcing compatibility between the informational and material projections.

In this form, the **Theory of Entropicity (ToE)** fulfills the exact demand stated earlier in Section 6 of this Letter III: it derives both sides of Einstein's elegant field equations from entropy, but only after entropy has been supplied with the required tensorial structure within the Entropic Field.

Now impose the Einstein sector of the theory: near equilibrium, weak gradient, reversible macroscopic limit, with

$$S = S_{\mathrm{eq}} + \delta S, \qquad X \ll 1, \qquad \alpha \to 0, \qquad q \to 1, \qquad H_{\mu\nu}^{\mathrm{corr}} \to 0, \qquad G_{\mathrm{eff}} \to G. \ (9.6.2)$$

For a **canonical coherent sector (CCS)** $F(X,S) = X - V(S)$,

$$T_{\mu\nu}^{(S)} = \nabla_\mu S \nabla_\nu S + \big(X - V(S)\big) g_{\mu\nu}. \ (9.6.3)$$

At equilibrium, $X \to 0$ and $S \to S_{\mathrm{eq}}$, so

$$T_{\mu\nu}^{(S)} \to -V\big(S_{\mathrm{eq}}\big)\, g_{\mu\nu} + \delta T_{\mu\nu}^{(S)}. \ (9.6.4)$$

We now **move the vacuum-like piece to the geometric side** by defining the **macroscopic cosmological constant (MCC)**

$$\Lambda := \Lambda_{\mathrm{ent}} + 8\pi G\, V\big(S_{\mathrm{eq}}\big), \ (9.6.5)$$

while the genuine matter content is

$$T_{\mu\nu}^{(\mathrm{m})} := \delta T_{\mu\nu}^{(S)} + T_{\mu\nu}^{(C)} + T_{\mu\nu}^{(\mathrm{flux})}. \ (9.6.6)$$

The result is exactly

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(\mathrm{m})}. \ (9.6.7)$$

The above Equation (9.9.7) is the Einstein field equations, obtained as the infrared, near-equilibrium, tensorially completed limit of the Theory of Entropicity (ToE). In this completed structure, the left-hand side is entropy as information geometry, and the right-hand side is entropy as localized, constrained, flowing, and stress-bearing embodiment built from the Entropic Field.

This also clarifies the ToE hierarchy developed earlier. **Jacobson** shows how **Einstein's** equation can be read as a thermodynamic equation of state. **Matsueda** shows that **Fisher geometry** can produce Einstein-like tensors. **Caticha** and collaborators show that **information geometry** can be made dynamical and can reproduce both **quantum mechanics** and **classical GR** in appropriate limits. **Bianconi** constructs an entropic action from relative entropy that reduces to Einstein dynamics at low coupling and small curvature.

The Theory of Entropicity (ToE) contribution, in the canonical form given here, is to integrate these strands into one program: Lorentzian emergence from entropy-directed information geometry for the left-hand side, and entropic moment/covariance completion for the right-hand side.

Thus, the mathematical claim of ToE is not that:

$$S \equiv T_{\mu\nu}, \quad (9.6.8)$$

which would be dimensionally and tensorially wrong and of no enduring physical utility. **What the Theory of Entropicity (ToE) declares is logically motivated, straight, and direct:**

$$\textbf{entropy field} \;\Rightarrow\; \textbf{Lorentzian information metric} \;\Rightarrow\; [\boldsymbol{physical}]\ \textbf{curvature tensor}, \quad (\mathbf{9.6.9})$$

and simultaneously that:

$$\textbf{entropy field} \;\Rightarrow\; \textbf{coherent storage + covariance + flux moments} \;\Rightarrow\; \boldsymbol{total\ entropic}\ \textbf{source tensor (TEST)}. \quad (\mathbf{9.6.10})$$

Thus, Einstein's General Relativity is recovered when both projections are taken in the same low-energy classical limit. That is the mathematically coherent ToE formulation of Einstein's beautiful field equations, and it is the rendering that makes full sense of the ToE program already laid down in Section 6 of this Letter III.

PART III

Irreversibility, Arrow of Time, and the Broader Landscape

10. The Vuli–Ndlela Integral: Irreversibility and the Arrow of Time

General Relativity is time-symmetric: if (M, $g_{\mu\nu}$) is a solution of Einstein's equations, so is the time-reversed version. This is a significant deficiency for a theory aspiring to describe the physical universe, which exhibits a robust macroscopic arrow of time. The Theory of Entropicity resolves this by incorporating irreversibility directly into the fundamental variational principle through the Vuli–Ndlela Integral.

10.1 Definition and Structure

The Vuli–Ndlela Integral is ToE's entropy-weighted reformulation of the Feynman path integral. It is defined as:

$$Z_{(ToE)} = \int \mathcal{D}\,S exp\Big(\big(i/\hbar_{eff}\big)\big[\mathcal{S}_{class}[S] + \mathcal{S}_G[S] + \mathcal{S}_{irr}[S]\big]\Big) \ (10.1)$$

where:

- $\mathcal{S}_{class}[S]$is the classical entropic action (the Obidi Action);
- $\mathcal{S}_G[S]$ encodes gravitational-entropy corrections (horizon area, curvature);
- $\mathcal{S}_{irr}[S]$ is the irreversibility functional, representing entropy production and time-directed dissipation;
- $\hbar_{eff}$ = ħ exp($-\mathcal{S}_{irr}$/kB) is the entropy-modified Planck constant.

The entropy-modified Planck constant $\hbar_{eff}$ introduces causal damping of time-reversed paths: configurations with positive entropy production ($\mathcal{S}_{irr}$ > 0) are weighted normally, while configurations with negative entropy production (that is, time-reversed paths, $\mathcal{S}_{irr}$ < 0) are exponentially suppressed. This is the microscopic origin of the arrow of time.

10.2 The Complexified Entropic Euler Equation of ToE

Variation of (9.1) with respect to S(x) yields a **complexified entropic Euler equation:**

$$\nabla_\mu \nabla^\mu S - \frac{\partial V}{\partial S} + \frac{i}{\hbar}\frac{\delta S_{irr}}{\delta S} = J(x) \ (10.2)$$

Decomposing S = S_R + iS_I into reversible (S_R) and irreversible (S_I) parts:

$$\nabla_\mu \nabla^\mu S_R - \frac{\partial V}{\partial S_R} = J(x) \quad \text{[reversible sector] (10.3)}$$

Or

$$\Box S_R - \frac{\partial V}{\partial S_R} = J(x) \quad \text{[reversible sector] (10.3.1)}$$

$$\nabla_\mu \nabla^\mu S_I - \frac{\partial V}{\partial S_I} = \frac{1}{\hbar}\frac{\delta S_{irr}}{\delta S_I} \quad \text{[irreversible sector] (10.4)}$$

Or

$$\Box S_I - \frac{\partial V}{\partial S_I} = \frac{1}{\hbar}\frac{\delta S_{irr}}{\delta S_I} \quad \text{[irreversible sector](10.4.1)}$$

The irreversible sector S_I governs dissipation and entropy production. Solutions with S_I > 0 correspond to forward-causal evolution; solutions with S_R < 0 represent retrocausal configurations exponentially suppressed by the $\hbar_{eff}$ factor in (9.1).

10.3 The No-Rush Theorem (NRT)

From the entropic current conservation (3.5), one derives a non-negativity condition:

$$\nabla_\mu J^\mu_{(ent)} \geq 0 \text{ (10.5)}$$

This states that the **covariant divergence of the entropic current is non-negative**, meaning the local production or outflow of entropy cannot be negative. In ToE language, it expresses the **irreversibility condition**: entropy flow may be conserved in an ideal reversible limit, but in the irreversible sector it satisfies a non-negative production law.

This is the Entropic No-Rush Theorem (NRT) of ToE: information and energy flow cannot outrun the causal clock of the entropy field. The rate of entropy production in any local region is bounded from below by zero.

This theorem subsumes the Second Law of Thermodynamics as a consequence of entropic field dynamics, rather than a statistical postulate.

10.4 Connection to General Relativity

In the limit $\mathcal{S}_{irr} \to 0$, the Vuli–Ndlela Integral reduces to the standard Feynman path integral with the Obidi Action. Further taking the classical limit $\hbar \to 0$ recovers the classical entropic field equations (7.4). And in the near-equilibrium, weak-gradient limit, these reduce to Einstein's field equations.

The connection to Bianconi's time-symmetric framework is:

$$\text{Bianconi Gravity from Entropy} \;=\; \text{ToE}\,|\_\{\mathcal{S}_{irr} = 0,\;\; \alpha = 1\} \qquad (10.6)$$

Bianconi's theory, working with a symmetric Kullback–Leibler divergence, is time-symmetric by construction. ToE extends this to the fully time-asymmetric theory by including $\mathcal{S}_{irr} \neq 0$.

11. ToE Within the Landscape of Entropic Gravity

The idea that gravity might be entropic in origin has a distinguished history. This section situates ToE within this landscape, demonstrating how each previous approach is contained within the ToE hierarchy as a special case.

11.1 Jacobson's Thermodynamic Derivation

In 1995, Ted Jacobson derived Einstein's field equations from the thermodynamic Clausius relation $\delta Q = T\, \delta S$ applied to local Rindler horizons. This was a landmark result: it showed that Einstein's equations are, in a precise sense, an equation of state of spacetime thermodynamics.

Within ToE, Jacobson's derivation corresponds to the limit where the entropy field is entirely determined by horizon geometry — the regime in which $S(x)$ is frozen to its equilibrium value determined by the area of local causal horizons. In this limit, the Master Entropic Equation reduces to the Clausius relation, and the Obidi Action reduces (via the Wald entropy formula) to the Einstein–Hilbert Action. Jacobson's result is thus a boundary condition on the full entropic field theory.

11.2 Verlinde's Emergent Gravity

Erik Verlinde's 2010 proposal derives Newtonian gravity and, in a more ambitious extension, the full Einstein equations from holographic entropy on codimension-1 screens. Verlinde treats entropy as a function of information content on these screens, and derives gravitational force as an entropic force — the tendency of the system to increase entropy.

Within ToE, Verlinde's approach corresponds to a surface/boundary limit of the bulk entropic field theory. The holographic screens are level surfaces of $S(x)$, and the entropic force is the normal gradient of the entropy field. The equipartition of energy on the screen follows from the equipartition of the quadratic fluctuations of δS. ToE provides the bulk field theory of which Verlinde's approach is the holographic boundary description.

11.3 Bianconi's Gravity from Entropy

Ginestra Bianconi's 2025 paper in Physical Review D constructs a gravitational action from the quantum relative entropy between the spacetime metric (treated as a density matrix) and a matter-induced metric. This is an elegant and technically sophisticated construction.

As demonstrated in the attached companion paper, Bianconi's action is exactly recovered from the Obidi Action in the near-equilibrium, weak-gradient, time-symmetric limit. The Obidi Action at quadratic order in δS, in the Fisher–Shannon/von Neumann sector ($\alpha = 1$, $q = 1$), with $S_{irr} = 0$, equals Bianconi's relative-entropy action. The G-field of Bianconi's theory emerges naturally as the Lagrange multiplier enforcing entropic consistency in ToE. The emergent cosmological constant Λ of Bianconi's theory equals $\Lambda_{ent} = \langle(\nabla S)^2\rangle$ in ToE.

The complete hierarchy is:

Theory / Approach	**ToE Limit**
Einstein GR (no entropy)	$\nabla S = 0$, $V'(S) = \text{const}$ (static entropy limit)
Jacobson thermodynamic derivation	$S(x)$ frozen at horizon geometry (boundary limit)
Verlinde emergent gravity	Holographic surface limit of bulk ToE
Padmanabhan entropy cosmology	Near-equilibrium, isotropic $S(x)$ with uniform gradients
Bianconi Gravity from Entropy	Quadratic ToE, $\alpha = 1$, $q = 1$, $S_{irr} = 0$
Full ToE (general (α,q), $S_{irr} \neq 0$)	Universal entropic field theory (parent theory)

Table 3: Embedding of previous entropic gravity approaches within the ToE hierarchy.

PART IV

Predictions, Implications, and Outlook

12. Distinguishing Predictions and Experimental Consequences

A theory that merely reproduces known results has limited scientific value. The power of ToE lies in its ability to make predictions that go beyond standard GR and its competitors. These predictions arise from the full ToE equations (7.4) — with the entropic stress-energy $T(S)_{\mu\nu}$, the dynamic Λ_{ent}, and the α-field — terms that vanish in the GR limit but are non-zero in the full theory.

12.1 A Dynamical Cosmological Constant

Perhaps the most significant prediction of ToE concerns the cosmological constant. In GR, Λ is a free parameter inserted by hand, with no dynamical principle determining its value. In ToE, Λ_{ent} is determined by:

$$\Lambda_{eff}(x, t) = \langle(\nabla S)^2\rangle + \xi [(\nabla S)^2 - SR] \quad (12.1)$$

where R is the Ricci scalar and ξ is the entropic curvature coupling. This predicts a small, spatially varying, and redshift-dependent correction to the cosmological constant — precisely the kind of deviation from ΛCDM that current and future cosmological surveys (Euclid, Roman Space Telescope, DESI) are designed to detect.

12.2 Entropic Lensing and Time-Delay Effects

In ToE, light propagation is affected not only by the geometric metric $g_{\mu\nu}$ but also by the entropy gradient. The null geodesic equation acquires an entropic correction:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = E^\mu \ (12.2)$$

Where $E^\mu = \eta\nabla^\mu S(\nabla S)^2$ [or $E^\mu = \eta\nabla^\mu S(\nabla_\alpha S\nabla^\alpha S)$] is the entropic deviation force. This produces additional focusing of light proportional to $(\nabla S)^2$, leading to modified lensing and Shapiro-type time delays:

$$\Delta t_{\text{ent}} \sim \int_{\text{path}} (\nabla S)^2 \, , d\ell \ (12.3)$$

Or

$$\Delta t_{\text{ent}} \sim \int_{\text{path}} \left(\nabla_\mu S \nabla^\mu S\right) d\ell \ (12.3.1)$$

In regions of strong entropy gradients — near massive black holes, in the early universe, or in regions of strong dark energy density — this correction could be observable via ultra-precise pulsar timing or strong-lensing analysis.

12.3 Quantum Coherence Modification

The entropy-modified Planck constant $\hbar_{eff} = \hbar\ \exp(-S_{irr}/kB)$ implies that quantum coherence is modulated by entropy production. In regions of high entropy production (irreversible processes, thermodynamic non-equilibrium), $\hbar_{eff}$ decreases, leading to enhanced decoherence. This predicts:

$$\hbar_{eff} < \hbar \quad \text{in regions of high entropy production} \quad (12.4)$$

Measurable via attosecond entanglement experiments or precision quantum interference measurements in high-temperature or high-dissipation environments.

12.4 The Obidi Curvature Invariant

ToE predicts a universal lower bound on the entropic cost of distinguishing two physical states — the **Obidi Curvature Invariant (OCI):**

$$\text{OCI} = \ln 2 \approx 0.693 \quad (12.5)$$

This represents the minimum threshold of distinguishability in the universe, arising from the minimum curvature of the entropic manifold. Below this threshold, two physical configurations cannot be distinguished by any measurement, no matter how precise. This provides a fundamental, information-theoretic resolution of the measurement problem in quantum mechanics.

Physical Phenomenon	ToE Prediction	Observable / Test
Cosmological acceleration	$\Lambda_{eff}(z)$ varies with redshift	SN Ia, BAO, CMB (Euclid, Roman)
Gravitational lensing	Extra focusing $\propto (\nabla S)^2$	Strong lensing, pulsar timing
Shapiro time delay	Entropic correction Δt_ent	VLBI, pulsar timing arrays
Quantum coherence	$\hbar_{eff} = \hbar$ e^(−Sirr/kB)	Attosecond entanglement experiments
CMB non-Gaussianity	Entropy-curvature correlations	CMB polarization surveys
Cosmological constant	$\Lambda = \langle(\nabla S)^2\rangle$ (dynamical)	DESI, Euclid large-scale structure
Decoherence enhancement	Entropy production → $\hbar_{eff}\downarrow$	Quantum optics, condensed matter

Table 4: Predicted observational consequences of ToE beyond standard GR.

13. Philosophical and Conceptual Implications

13.1 The Dissolution of the Geometry–Thermodynamics Divide

One of the longest-standing conceptual divides in theoretical physics is the separation between geometry (which GR takes as fundamental and deterministic) and thermodynamics (which statistical mechanics treats as emergent and probabilistic). ToE dissolves this divide by revealing that both are aspects of a single entropic substrate:

- Geometry is the large-scale projection of entropy gradients onto a macroscopic background.
- Thermodynamics is the local statistical manifestation of entropy flux in the entropic manifold.
- Gravitation is the dynamical consequence of entropy conservation and redistribution.

In this unified picture, the famous "unreasonable effectiveness of mathematics" in physics finds a new explanation: mathematics is effective because the universe is not made of matter or geometry, but of information — and mathematics is, at its core, the science of information structure.

13.2 The Self-Computing Universe

A central philosophical implication of ToE is the concept of the self-computing universe. The solutions of the Obidi Field Equations are iterative rather than explicit — the universe does not occupy a fixed point in configuration space but continuously "computes" its next state through the dynamics of the entropy field. The universe is not a machine running on fixed laws; it is an ongoing computation, constantly recalculating its own state through the irreversible dynamics of entropic reorganization.

This connects ToE to information-theoretic interpretations of quantum mechanics (Zeilinger, Wheeler's "it from bit") and to the computational universe hypothesis (Wolfram, Lloyd), but with a precise mathematical framework — the Obidi Action — that the latter lack.

13.3 Entropy as Ontological Substrate

Perhaps the deepest philosophical claim of ToE is the ontological primacy of entropy. In all previous physics, entropy has been a secondary or derived quantity. In ToE, it is the primary substrate of reality — the "stuff" from which spacetime, matter, fields, and forces are made.

This represents a third great conceptual inversion in the history of physics, comparable to Newton's inversion of Aristotelian teleology (replacing final causes with mechanical laws) and Einstein's inversion of gravitational force (replacing Newtonian attraction with geometric curvature). Obidi's inversion replaces geometric primacy with entropic primacy — not as a metaphysical claim, but as a precise mathematical thesis with testable consequences.

14. Conclusion and Summary

This monograph has established, with mathematical rigor and conceptual precision, the correspondence between the **Obidi Action** of the **Theory of Entropicity (ToE)** and the **Einstein–Hilbert Action** of **General Relativity (GR)**. The central results may be summarized as follows:

1. The Obidi Action is the fundamental variational principle of ToE, governing the dynamics of the ontological entropy field S(x). It occupies the same structural role in ToE that the Einstein–Hilbert Action occupies in GR.
2. The Einstein–Hilbert Action is recovered from the Obidi Action in the near-equilibrium, weak-gradient, large-scale, Boltzmann–Gibbs ($\alpha = 0$, $q = 1$) limit. Einstein's field equations are thus a special case of the entropic field equations of ToE.
3. The Fisher–Rao metric, Fubini–Study metric, and Amari–Čencov α-connections constitute the information-geometric foundation from which physical spacetime emerges. The Levi–Civita connection of GR is the effective, emergent affine connection that arises when the α-connections reduce in the classical limit.
4. The constitutive constraint $\alpha = 2(1-q)$ links the Rényi–Tsallis non-extensivity parameter to the affine asymmetry of the α-connections, embedding both within the unified Obidi Action dynamics.
5. The Vuli–Ndlela Integral introduces irreversibility and the arrow of time into the framework, extending GR's time-symmetric structure to a fully causal, entropic field theory.
6. The entropic cosmological term $\Lambda_{ent} = \langle(\nabla S)^2\rangle$ is dynamically determined by the variance of entropy gradients, rather than being a free parameter — resolving in principle the cosmological constant problem.
7. The ToE hierarchy Einstein GR ⊂ Bianconi Entropic Gravity ⊂ Obidi's Theory of Entropicity unifies all previous entropic approaches to gravity as limiting cases of the same universal entropic field theory.

The Theory of Entropicity (ToE) does not contradict or refute previous physics. It subsumes it, completes it, and reveals its deeper information-geometric foundation. Where Einstein showed that geometry is physics, and where Jacobson, Verlinde, and Bianconi showed that thermodynamics and information are geometry, Obidi shows that information geometry is the origin of geometry itself — and therefore of gravity, spacetime, and physical reality.

The universe, in this understanding, does not merely evolve within spacetime. It continuously computes spacetime through the irreversible dynamics of an entropy field whose gradients, fluxes, and conservation laws generate everything that physics has previously taken as given. The speed of light is the entropic speed limit. Gravity is the macroscopic shadow of information-geometric curvature. Time is the direction of increasing entropy. And at the foundation of it all stands the Obidi Action — the universal law of entropic becoming.

"Where Einstein revealed the constancy of light as the foundation of spacetime, and Bianconi revealed relative entropy as a generator of gravity, the Theory of Entropicity (ToE) reveals entropy itself as the ontic substrate of the universe — and hence the foundation of nature and reality."

— John Onimisi Obidi, Theory of Entropicity (ToE), 2025

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PART V

Mathematical Appendices and Supplementary Materials

A. Mathematical Appendices and Supplementary Materials

The following appendices present the key results, derivations, and conceptual clarifications that underpin the Theory of Entropicity (ToE). They gather in one place the essential mathematical structures, reformulations, and interpretive insights required to understand why the ToE program is not optional but **logically necessary** for recovering the Einstein Field Equations of General Relativity (GR) from first principles.

Each appendix develops a different facet of the entropic foundations of spacetime—ranging from the construction of the **Obidi Action**, to the emergence of **information geometry**, to the reinterpretation of matter, energy, and curvature as dual manifestations of a single entropic substrate. Together, these results demonstrate how the axioms and variational principles of ToE give rise to both sides of Einstein's equation:

- the **geometric sector** (the left-hand side), arising from entropic information curvature, and
- the **source sector** (the right-hand side), emerging from structured, constrained, and dynamically evolving information entropy.

By organizing these developments into appendices, the main text remains focused on the conceptual narrative, while the technical foundations are presented with full transparency and rigor. The reader is thus equipped to follow the complete logical chain: from the primitive entropic axioms, through the construction of the entropic manifold, to the derivation of the effective gravitational dynamics that reproduce GR in the appropriate limit.

These appendices therefore serve not merely as supplementary material, but as the **mathematical engine room** of the theory—showing explicitly how ToE transforms entropy, information, and geometric structure into a unified framework capable of generating the Einstein Field Equations from first principles of establishing entropy as a universal substrate.

A.1 Speed of Light c as the Entropic Speed Limit

In ToE, the universal constant c is reinterpreted as:

$$c \equiv \max\left(\frac{dS}{dt}\right), (A.1.1)$$

the maximum rate at which the entropy field can reorganize matter/mass/stress/energy and information. This implies that Lorentz invariance is not a geometric postulate but a theorem of entropic dynamics. Light cones, simultaneity, time dilation, and mass increase all follow from the constraint that entropic reorganization cannot exceed c.

A.2 Master Entropic Equation (MEE)

Variation with respect to $S(x)$yields:

$$\kappa_S \nabla_\mu \nabla^\mu S - \frac{dV}{dS} + \Lambda_S(S, \nabla S, g) = 0, (A.2.1)$$

or compactly:

$$\Box_g S - \frac{1}{\kappa_S}\frac{dV}{dS} = J(x). (\mathrm{A}.2.2)$$

This is the entropic analogue of the Klein–Gordon and Einstein equations.

A.3 Entropic Current and Conservation

Define the entropic current:

$$J^\mu = \eta \nabla^\mu S, (A.3.1)$$

with conservation law:

$$\nabla_\mu J^\mu = 0. (A.3.2)$$

This enforces global preservation of entropy flux, analogous to charge conservation.

A.4 Entropic Stress–Energy Tensor

Variation with respect to $g_{\mu\nu}$yields:

$$T_{\mu\nu}^{(S)} = \nabla_\mu S \nabla_\nu S - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha S)(\nabla^\alpha S) + g_{\mu\nu} V(S) - g_{\mu\nu} J(x) S. (\mathrm{A}.4.1)$$

This tensor sources emergent gravity, playing the role of matter stress–energy in GR.

A.5 The Mathematical Correspondence

5.1 The Einstein–Hilbert Action: Structure and Role

Before establishing the correspondence, it is useful to recall the structure of the Einstein–Hilbert action and its role in General Relativity (GR), so that the structural parallels with the Obidi Action can be made precise.

5.2 The Einstein–Hilbert Action

$$S_{\mathrm{EH}}[g] = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, R[g] + S_{\mathrm{M}}[\Phi, g], (5.2.1)$$

where:

- G_N is Newton's gravitational constant,
- $R[g] = g^{\mu\nu} R_{\mu\nu}$is the Ricci scalar,
- $S_{\mathrm{M}}[\Phi, g]$ is the matter action, coupling matter fields Φ to the metric $g_{\mu\nu}$,
- $\sqrt{-g}\, d^4x$ is the invariant volume element.

Variation with respect to $g_{\mu\nu}$ yields Einstein's field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}^{(\mathrm{M})}, (5.2.2)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor, Λ is the cosmological constant, and $T_{\mu\nu}^{(\mathrm{M})}$is the matter stress–energy tensor.

5.3 Emergence Map

Define a coarse-graining map:

$$\Phi: \mathcal{M}_4 \longrightarrow \mathcal{M}_{\mathrm{info}}, x^\mu \mapsto \theta^a(x). (5.3.1)$$

The pullback Φ^*acts on scalar densities and curvature scalars on $\mathcal{M}_{\mathrm{info}}$to produce fields on $\mathcal{M}_4$.

Pullback of the metric: construct the emergent spacetime metric − Pullback relation for the metric

The emergent spacetime metric is the pullback:

$$g_{\mu\nu}(x) = \lambda^2\ \partial_\mu \theta^a(x)\ \partial_\nu \theta^b(x)\, G_{ab}(\theta(x)). (5.3.2)$$

Consequence

This defines $g_{\mu\nu}(x)$as a smooth 4D metric field on $\mathcal{M}_4$built from $G_{ab}$and the embedding functions $\theta^a(x)$.

5.4 Measure Transfer

$$\Phi^* \square \left(\sqrt{|\, G\, |}\ d^n\theta\right) = \mathcal{J}(x) \sqrt{-g}\, d^4x, (5.4.1)$$

where $\mathcal{J}(x)$ is the Jacobian density from projecting the entropic information manifold onto its macroscopic four-dimensional sector.

5.5 Curvature Transfer

$$\Phi^{*}(\mathcal{R}[G]) = R[g] + \Delta_{\text{extr}} + \Delta_{\text{cg}}, (5.5.1)$$

where Δ_{extr}encodes extrinsic curvature corrections and Δ_{cg}encodes coarse-graining corrections. In the infrared limit:

$$\Delta_{\text{extr}} + \Delta_{\text{cg}} \to 0, \mathcal{J}(x) \to Z_G. (5.5.2)$$

5.6 Action Correspondence

The gravitational sector of the Obidi Action becomes:

$$S_{\text{Obidi,grav}}^{\text{IG}} = \frac{1}{16\pi G_I}\int_{\mathcal{M}_{\text{info}}} d^n\,\theta\,\sqrt{|\,G\,|}\;\mathcal{R}[G] \quad\longrightarrow\quad \frac{Z_G}{16\pi G_I}\int_{\mathcal{M}_4} d^4\,x\,\sqrt{-g}\,R[g]. (5.6.1)$$

Identifying:

$$\frac{1}{G_N} = \frac{Z_G}{G_I}, (5.6.2)$$

we recover:

$$S_{\text{Obidi,grav}}^{\text{IG}} \longrightarrow S_{\text{EH}}[g]. (5.6.3)$$

Thus, the Einstein–Hilbert action is the macroscopic manifestation of information geometry.

5.7 Field Equations

At the level of equations of motion:

$$\mathcal{G}_{ab}[G] = 8\pi G_I\,\mathcal{T}_{ab}^{(\text{ent})} \Longrightarrow G_{\mu\nu}[g] = 8\pi G_N\,T_{\mu\nu}^{\text{eff}} + \Lambda_{\text{eff}} g_{\mu\nu} + O(\Delta). (5.7.1)$$

5.8 Pullback of the measure: Jacobian density

Pullback of the volume form

Under Φ, the information-manifold volume element transforms as

$$\Phi^{*}(\sqrt{|\,G(\theta)\,|}\;d^n\theta) = \mathcal{J}(x)\,\sqrt{-g(x)}\,d^4x, (5.8.1)$$

where $\mathcal{J}(x)$is the Jacobian density produced by projecting the n-dimensional information volume onto the 4D image of Φ.

Leading-order approximation in the IR

Assume that on macroscopic scales the projection is approximately homogeneous so that

$$\mathcal{J}(x) = Z_G + \delta\mathcal{J}(x), \delta\mathcal{J}(x) \ll Z_G, (5.8.2)$$

and in the infrared limit $\delta\mathcal{J}(x) \to 0$, leaving the constant factor Z_G.

Pullback / expansion of the curvature scalar

Formal pullback

$$\Phi^* (\mathcal{R}[G])(x) = \mathcal{R}[G(\theta(x))] |_{\theta=\theta(x)}. (5.8.3)$$

Geometric expansion

Express the pulled-back information curvature in terms of the emergent spacetime curvature plus corrections:

$$\Phi^* (\mathcal{R}[G]) = R[g] + \Delta_{\text{extr}} + \Delta_{\text{cg}}. (5.8.4)$$

- Δ_{extr}collects terms arising from extrinsic curvature of the 4D image inside the information manifold and from derivatives of $\partial_\mu \theta^a$.
- Δ_{cg}collects short-distance, high-frequency, and nonlocal coarse-graining corrections.

Assumption for IR limit

On macroscopic scales and after smoothing/coarse-graining:

$$\Delta_{\text{extr}} + \Delta_{\text{cg}} \longrightarrow 0. (5.8.5)$$

This is the controlled approximation that the information curvature projects dominantly onto the intrinsic 4D Ricci scalar.

Substitute pullbacks into the action

Start from the left action and apply the pullbacks for measure and curvature:

$$S_{\text{Obidi,grav}}^{\text{IG}} = \frac{1}{16\pi G_I} \int_{\mathcal{M}_{\text{info}}} d^n\,\theta \sqrt{|\,G\,|}\ \ \mathcal{R}[G]$$
$$= \frac{1}{16\pi G_I} \int_{\mathcal{M}_4} d^4\,x\ \Phi^* (\sqrt{|\,G\,|}\ \ \mathcal{R}[G])$$
$$= \frac{1}{16\pi G_I} \int_{\mathcal{M}_4} d^4\,x\ \mathcal{J}(x) \sqrt{-g(x)}\ \ (R[g(x)] + \Delta_{\text{extr}}(x) + \Delta_{\text{cg}}(x)). (5.8.6)$$

Infrared limit and identification of constants

Take the IR limit where corrections are negligible and $\mathcal{J}(x) \to Z_G$:

$$S_{\text{Obidi,grav}}^{\text{IG}} \longrightarrow \frac{Z_G}{16\pi G_I} \int_{\mathcal{M}_4} d^4\,x \sqrt{-g}\ \ R[g]. (5.8.7)$$

Define the effective Newton constant

Identify the emergent Newton coupling G_N by

$$\frac{1}{G_N} \equiv \frac{Z_G}{G_I} \Longrightarrow \frac{Z_G}{16\pi G_I} = \frac{1}{16\pi G_N}. \quad (5.8.8)$$

Final result

$$S_{\text{Obidi,grav}}^{\text{IG}} \xrightarrow{\text{IR, coarse-graining}} \frac{1}{16\pi G_N}\int_{\mathcal{M}_4} d^4x\,\sqrt{-g}\;R[g] = S_{\text{EH}}[g]. \quad (5.8.9)$$

Summary of assumptions and leading corrections

Key assumptions used

- Existence of a smooth embedding $\theta^a(x)$with nondegenerate Jacobian on the macroscopic sector.
- Homogenization of the Jacobian density so $\mathcal{J}(x) \to Z_G$in the IR.
- Controlled suppression of extrinsic and coarse-graining corrections: $\Delta_{\text{extr}} + \Delta_{\text{cg}} \to 0$at large scales.
- Scale factor λabsorbed into the definition of $Z_G$or into $G_I$as needed.

Leading corrections (kept track of symbolically)

If corrections are retained, the emergent action reads

$$S_{\text{emergent}} = \frac{Z_G}{16\pi G_I}\int d^4x\,\sqrt{-g}\;R[g] + \frac{1}{16\pi G_I}\int d^4x\,\sqrt{-g}\;\big(\delta\mathcal{J}(x)\,R[g] + \mathcal{J}(x)\,\Delta_{\text{extr}} + \mathcal{J}(x)\,\Delta_{\text{cg}}\big). \quad (5.8.10)$$

These terms produce higher-derivative, nonlocal, or matter-coupled corrections to Einstein gravity and are the natural place to look for phenomenology distinguishing the Theory of Entropicity (ToE) from pure General Relativity (GR).

Concluding remark

The arrow

$$\frac{1}{16\pi G_I}\int_{\mathcal{M}_{\text{info}}} \sqrt{|\,G\,|}\;\mathcal{R}[G] \;\longrightarrow\; \frac{Z_G}{16\pi G_I}\int_{\mathcal{M}_4}\sqrt{-g}\;R[g] \quad (5.8.11)$$

is obtained by **(i)** pulling back metric and curvature via $\theta^a(x)$, **(ii)** transferring the measure with Jacobian $\mathcal{J}(x)$, **(iii)** expanding the pulled-back curvature into intrinsic plus extrinsic/coarse-graining pieces, and **(iv)** taking the infrared coarse-grained limit in which $\mathcal{J} \to Z_G$and correction terms are negligible, then identifying $G_N^{-1} = Z_G/G_I$.

A.6 The Obidi Transformation and the Obidi Metric of ToE

The **Obidi Transformation** and the resulting **Obidi Metric** constitute the central mathematical mechanism by which the Theory of Entropicity (ToE) converts the **positive-definite Fisher–Rao information metric** of statistical geometry into an **emergent Lorentzian metric** with one negative and three positive eigenvalues. This signature change is not a cosmetic modification but a **structural necessity**: it is the step that allows a purely Riemannian information manifold to become the **Einsteinian spacetime** required by General Relativity (GR).

In standard information geometry, the Fisher–Rao metric encodes distinguishability between probability distributions and is therefore strictly Riemannian. However, physical spacetime is not Riemannian; it is **Lorentzian**, with a built-in causal structure, light cones, and a distinguished timelike direction. The Obidi Transformation provides the **unique entropic deformation** that identifies the entropy gradient as the preferred temporal direction and flips exactly one eigenvalue of the information metric. The resulting Obidi Metric is thus the **minimal rank-one modification** that converts information geometry into a physically meaningful spacetime geometry.

This appendix develops the transformation in full detail, showing how the entropy field selects a canonical timelike vector, how the deformation preserves the essential information-geometric structure, and how the resulting Lorentzian metric becomes the geometric backbone of the emergent gravitational dynamics. The construction demonstrates that the signature change is not imposed by hand but arises from the **entropic axioms** of ToE, making the emergence of spacetime a **logical consequence** of the theory rather than an assumption.

By presenting the derivation, properties, and implications of the Obidi Transformation and Obidi Metric, this appendix clarifies why these constructs are indispensable for the ToE program. They supply the mathematical bridge between **information-theoretic structure** and **Einsteinian geometry**, enabling ToE to reproduce the spacetime manifold, causal structure, and curvature dynamics of GR from first principles.

1. Setup and guiding idea

Guiding idea. Use the entropy field $S(\theta)$(or its spacetime image $S(x)$after Λ□ → □x) to single out a preferred direction on the information manifold and flip one metric eigenvalue. Concretely: the entropy gradient defines a canonical vector field; use that vector to construct a rank-one deformation of the Fisher–Rao metric that produces one negative eigenvalue while preserving spatial positive definiteness.

Assumptions.

- The information manifold $\mathcal{M}_{\text{info}}$carries the Fisher–Rao metric $G_{ab}(\theta)$, positive definite.

- The entropy field $S(\theta)$is smooth and has **nonzero gradient** on the macroscopic sector: $\nabla_a S \neq 0$.
- The image of the emergence map $\Phi$is a smooth 4D submanifold where these constructions are well defined.
- We work locally where $\nabla_a S$is timelike with respect to the new metric to be constructed.

2. Define the entropy gradient vector and its normalization

Entropy gradient vector

$$v_a(\theta) \equiv \nabla_a S(\theta). \text{(A. 6.1)}$$

Norm with respect to Fisher–Rao

$$\| v \|_G^2 \equiv G^{ab} v_a v_b > 0 \text{(since } G \text{ is Riemannian).}$$

Unit entropy direction

$$u_a \equiv \frac{v_a}{\sqrt{G^{cd} v_c v_d}}, u^a = G^{ab} u_b, G^{ab} u_a u_b = 1. \text{(A. 6.2)}$$

Interpretation. $u^a$is the unit vector field pointing along the local direction of steepest entropic change.

3. Construct the Obidi metric deformation that flips signature

Rank-one sign flip

Define the **Obidi metric** $\tilde{G}_{ab}$ by a rank-one deformation of G_{ab}:

$$\boxed{\tilde{G}_{ab} = G_{ab} - 2\, u_a u_b = G_{ab} - 2\, \frac{\nabla_a S\, \nabla_b S}{G^{cd} \nabla_c S\, \nabla_d S}. \text{(A. 6.3)}}$$

Eigenvalue check. Because $G_{ab}$is positive definite and $u^a$is unit, $\tilde{G}_{ab}$has one eigenvalue equal to -1in the u^adirection and the remaining $n-1$eigenvalues remain positive (shifted by the rank-one term). Thus $\tilde{G}_{ab}$has Lorentzian signature $(-,+,\dots,+)$.

Alternative conformal form. One may equivalently write a conformalized **Obidi metric** with a scale function $\Omega(S) > 0$:

$$\tilde{G}_{ab} = \Omega(S)(G_{ab} - 2\, u_a u_b). \text{(A. 6.4)}$$

The conformal factor Ω is useful to control curvature scales and to match units when pulling back to spacetime.

4. Interpret the time coordinate and foliation

Entropy time coordinate. Define a local "entropy time" function $t(\theta)$by integrating along integral curves of u^a:

$$\frac{d\theta^a}{dt} = u^a(\theta), t(\theta) = \int u_a \, d\theta^a. \text{(A. 6.5)}$$

Foliation. Level sets $\Sigma_t = \{\theta: t(\theta) = \text{const}\}$are $(n-1)$-dimensional spacelike hypersurfaces with respect to $\tilde{G}_{ab}$. The vector $u^a$is normal to these hypersurfaces and is timelike for $\tilde{G}$.

Physical meaning. Entropy gradients define a preferred time direction; **the Obidi metric makes that direction timelike.**

5. Relation between determinants and volume forms

Determinant identity for rank-one update. For a rank-one deformation $A_{ab} = G_{ab} + \alpha\, w_a w_b$, there is a standard determinant formula. Applied to $\tilde{G}_{ab} = G_{ab} - 2u_a u_b$,

$$\det\ \tilde{G} = \det\ G \cdot (1 - 2\, u^a u_a). \text{(A. 6.6)}$$

Using $u^a u_a = 1$(with indices raised by G^{ab}), we get

$$\det\ \tilde{G} = -\det\ G. \text{(A. 6.7)}$$

Thus $\sqrt{|\det\ \tilde{G}\ |} = \sqrt{\det G}$. The sign flip in the determinant is the algebraic signature change; the absolute value of the volume element is preserved at leading order.

6. Levi-Civita connection and curvature of the Obidi metric

Christoffel symbols relation. Let $\tilde{\nabla}$ be the Levi-Civita connection of $\tilde{G}$ and ∇ that of G. The **difference tensor** $C^a{}_{bc}$ is

$$C^a{}_{bc} = \tilde{\Gamma}^a{}_{bc} - \Gamma^a{}_{bc} = \frac{1}{2}\, \tilde{G}^{ad}(\nabla_b \tilde{G}_{dc} + \nabla_c \tilde{G}_{db} - \nabla_d \tilde{G}_{bc}). \text{(A. 6.8)}$$

Substitute the **Obidi Transformation** $\tilde{G}_{ab} = G_{ab} - 2u_a u_b$ and expand; terms involve $\nabla_a u_b$ and $\nabla_a \nabla_b S$. Explicitly:

$$C^a{}_{bc} = -\,\tilde{G}^{ad}(u_d \nabla_b u_c + u_d \nabla_c u_b - u_b \nabla_c u_d - u_c \nabla_b u_d), \text{(A. 6.9)}$$

up to symmetric rearrangement. These expressions are algebraic and can be expanded term by term for curvature computations.

Ricci scalar relation (schematic). The Ricci scalar $\mathcal{R}[\tilde{G}]$can be written as

$$\mathcal{R}[\tilde{G}] = \mathcal{R}[G] + \Delta_{\text{Obidi}}[S, G], \text{(A. 6.10)}$$

where Δ_{Obidi} collects terms built from u^a, $\nabla_a u_b$, and second derivatives $\nabla_a\nabla_b S$. To leading order in slow-variation (small $\nabla\nabla S$ relative to curvature scales), $\mathcal{R}[\tilde{G}]$projects onto the intrinsic curvature of the spatial slices plus controlled correction terms.

7. Pullback to emergent spacetime and matching conditions

Emergence map. Compose the **Obidi metric** with the emergence map $\Phi: x^\mu \mapsto \theta^a(x)$. The emergent spacetime metric is

$$g_{\mu\nu}(x) \;=\; \Phi^*(\tilde{G}_{ab}) \;=\; \lambda^2\, \partial_\mu\theta^a\, \partial_\nu\theta^b\, \left(G_{ab}(\theta) - 2\,\frac{\nabla_a S\,\nabla_b S}{G^{cd}\nabla_c S\,\nabla_d S}\right). \quad (\mathrm{A.6.11})$$

Signature check on spacetime. Provided $\partial_\mu\theta^a$maps the entropy direction into a timelike direction on the 4D image, $g_{\mu\nu}$has Lorentzian signature $(-,+,+,+)$.

Matching of actions. Use the determinant and curvature relations from steps 5–6 to show that the information action built from $\mathcal{R}[\tilde{G}]$pulls back to an Einstein–Hilbert form plus controlled corrections (extrinsic curvature and higher-derivative terms). The same coarse-graining and Jacobian arguments used earlier apply, with $\tilde{G}$ replacing G.

8. Conditions, limitations, and physical interpretation

Necessary conditions

- **Nonvanishing entropy gradient**: $\nabla_a S \neq 0$ on the macroscopic sector.
- **Smooth foliation**: integral curves of u^a define a global foliation (or at least local foliation) so that level sets are spacelike.
- **Slow variation**: second derivatives $\nabla_a\nabla_b S$ are small compared with curvature scales where the Einstein limit is sought, so correction terms are suppressed.

Limitations

- Where $\nabla_a S = 0$ (equilibrium points), the construction degenerates; signature change loci require separate treatment.
- Global topology or singularities may obstruct a global **Obidi foliation**; one then works patchwise.
- The rank-one flip is a classical construction; quantum corrections to signature emergence require additional analysis (operator ordering, path integral measure).

Physical interpretation

- The entropy gradient defines a **physical arrow of time** and a preferred timelike direction.

- The Obidi metric is the minimal geometric operation that converts statistical distinguishability (Riemannian) into causal spacetime (Lorentzian) by making the entropy flow timelike.
- Extrinsic and coarse-graining corrections encode departures from classical GR and are natural phenomenological signatures of ToE.

9. Compact statement of the Obidi Transformation

$$\textbf{Obidi Transformation}: G_{ab}(\theta) \overset{S(\theta)}{\rightarrow} \tilde{G}_{ab}(\theta) = G_{ab}(\theta) - 2\frac{\nabla_a S(\theta)\,\nabla_b S(\theta)}{G^{cd}(\theta)\,\nabla_c S(\theta)\,\nabla_d S(\theta)}. \quad (A.6.12)$$

Under the emergence map $\Phi: x^\mu \mapsto \theta^a(x)$ and in the infrared coarse-grained limit, $\Phi^*(\tilde{G})$defines a Lorentzian spacetime metric $g_{\mu\nu}$whose Einstein–Hilbert action is the leading macroscopic image of the information-gravity action built from $\mathcal{R}[\tilde{G}]$.

10. Worked example (local coordinates)

Choose local coordinates on $\mathcal{M}_{\text{info}}$with θ^0aligned with S so that $S = S(\theta^0)$. Locally $u_a \propto d\theta^0$. Then

$$G_{ab} = \begin{pmatrix} G_{00} & G_{0i} \\ G_{i0} & G_{ij} \end{pmatrix}, \tilde{G}_{ab} = \begin{pmatrix} G_{00} - 2 & G_{0i} \\ G_{i0} & G_{ij} \end{pmatrix}, (A.6.13)$$

so that the 00-component flips sign when $G_{00} - 2 < 0$. After pullback and rescaling, this yields a local metric with one negative eigenvalue. This coordinate flip is useful when implementing the **Obidi Transformation** in explicit models or numerical simulations.

A.6.1 Expansions of the Components of the Obidi Metric and Substitution in the Ricci Scalar

6.1 Ricci scalar difference formula

The Ricci scalars of two connections related by $C^a{}_{bc}$satisfy the exact identity

$$\tilde{\mathcal{R}} = \mathcal{R} + G^{bc}(\nabla_a C^a{}_{bc} - \nabla_b C^a{}_{ac}) + G^{bc}(C^a{}_{ad} C^d{}_{bc} - C^a{}_{bd} C^d{}_{ac}), (6.1.1)$$

where indices are raised with G^{ab}. Substituting the expression for $C^a{}_{bc}$and expanding gives the explicit curvature correction $\Delta_{\text{Obidi}} = \tilde{\mathcal{R}} - \mathcal{R}$.

6.2 Expanded form of $\Delta_{Obidi}[S, G]$

Carrying out the substitution and simplifying (grouping total divergences and quadratic derivative terms) yields the following exact, fully expanded expression:

$$\boxed{\Delta_{\text{Obidi}}[S, G] = -A - B.} \quad (6.2.1)$$

$$\boxed{\Delta_{\text{Obidi}}[S,G] \;=\; -A \quad -2\,(\nabla_a u_b)(\nabla^a u^b) + 2\,(\nabla_a u^a)^2 + 2\,u^a u^b\,\mathcal{R}_{ab}[G].\;(6.2.2)}$$

$$\boxed{\Delta_{\text{Obidi}}[S,G] \;=\; -\;2\,\nabla_a \,(u^a\,\nabla_b u^b) - 2\,\nabla_a \,(u^b\,\nabla_b u^a) \quad -B.\;(6.2.3)}$$

Equivalently, grouping divergence and non-divergence pieces,

$$\Delta_{\text{Obidi}}[S,G] = -2\,\nabla_a \,(u^a\nabla_b u^b + u^b\nabla_b u^a) - 2(\nabla_a u_b \nabla^a u^b - (\nabla_a u^a)^2) + 2\,u^a u^b \mathcal{R}_{ab}[G].\,(6.2.4)$$

Both boxed forms are algebraically equivalent and are convenient for different purposes:

- the **divergence form** is useful when integrating by parts inside an action (it produces boundary terms);
- the **non-divergence quadratic form** isolates the local correction built from gradients of u.

6.3 Rewriting the Obidi Metric in terms of S

Recall $u_a = \nabla_a S/\sqrt{G^{cd}\nabla_c S \nabla_d S}$. Substituting this relation everywhere gives Δ_{Obidi}explicitly in terms of S and G. For compactness we denote $N \equiv \sqrt{G^{cd}\nabla_c S \nabla_d S}$. Then $u_a = \nabla_a S/N$ and thus:

$$\nabla_a u_b \quad = \frac{\nabla_a \nabla_b S}{N} - \frac{\nabla_b S}{N^3}\,(\nabla_a \nabla_c S\,\nabla^c S),\ \nabla_a u^a \quad = \frac{\Box_G S}{N} - \frac{\nabla^a S\,\nabla^b S\,\nabla_a \nabla_b S}{N^3},\,(6.3.1)$$

and each term in Δ_{Obidi}can be written by substitution. The result is lengthy but explicit; so, schematically, we write:

$$\Delta_{\text{Obidi}}[S,G] = \frac{(\text{second derivatives of } S)^2}{N^2} + \frac{(\text{third derivatives of } S)\cdot\nabla S}{N^3} + \frac{u^a u^b \mathcal{R}_{ab}[G]}{1} - \;2\,\nabla_a(\cdots),\,(6.3.2)$$

with the divergence terms $\nabla_a(\cdots)$ given above.

6.4 Remarks and usage of the Obidi Metric

- The expression is **exact** (no small-quantity expansion assumed).
- The divergence terms integrate to boundary contributions in the action; the non-divergence quadratic terms produce local corrections to the Einstein–Hilbert image.
- The final term $2\,u^a u^b \mathcal{R}_{ab}[G]$shows that information-manifold Ricci curvature projected along the entropy direction contributes directly to the scalar correction.
- In the **slow-variation / IR limit** (second derivatives of S are small compared with curvature scales) the quadratic derivative terms are suppressed and Δ_{Obidi} reduces dominantly to

divergence + projection terms; this is the regime in which $\mathcal{R}[\tilde{G}]$pulls back directly to $R[g]$with small corrections.

6.5 Result of Expansions of the Obidi Metric and Substitution in the Ricci Scalar

$$\mathcal{R}[\tilde{G}] = \mathcal{R}[G] - 2\nabla_a(u^a\nabla_b u^b + u^b\nabla_b u^a) - 2(\nabla_a u_b\nabla^a u^b - (\nabla_a u^a)^2) + 2u^a u^b\mathcal{R}_{ab}[G]. \quad (6.5.1)$$

Substitute $u_a = \nabla_a S/\sqrt{G^{cd}\nabla_c S\nabla_d S}$ to express every term in $S$and G.

A.7.1 Scholium on Δ_{Obidi} the Curvature-Correction Term

7.1 The Meaning of Δ_{Obidi}

$\Delta_{\text{Obidi}}$is the **curvature-correction term** that measures the difference between:

- the Ricci scalar of the **Obidi Metric**, and
- the Ricci scalar of the **original Fisher–Rao information metric**.

In other words:

$$\Delta_{\text{Obidi}}[S,g] \equiv R(G_{\mu\nu}^{(\text{Obidi})}) - R(g_{\mu\nu}) \quad (7.1.1)$$

It tells you **exactly how much curvature is added** when you apply the **Obidi Transformation** to the Fisher–Rao metric.

7.2 Why Δ_{Obidi} Exists

The **Obidi Transformation** modifies the metric by adding a rank-one term built from the entropy gradient:

$$G_{\mu\nu}^{(\text{Obidi})} = g_{\mu\nu} - 2\,\frac{\nabla_\mu S\nabla_\nu S}{g^{\alpha\beta}\nabla_\alpha S\nabla_\beta S} \quad (7.2.1)$$

This deformation:

- flips one eigenvalue
- creates a Lorentzian signature
- introduces a preferred entropic direction
- changes the Levi-Civita connection
- and therefore changes the curvature

Δ_{Obidi} captures **all** of that change above.

7.3 What Δ_{Obidi} Represents Physically

Δ_{Obidi} is the **entropic curvature contribution** — the part of the Ricci scalar that comes *purely* from the entropy-gradient deformation.

It is the mathematical object that makes the Obidi Action:

$$A_{\text{Obidi}} = \int d^4x\sqrt{-G}\, R(G)$$

reduce to the Einstein–Hilbert action in the emergent limit.

7.4 Interpretation of Δ_{Obidi} inside ToE

Inside the Theory of Entropicity (ToE):

- $g_{\mu\nu}$= raw information geometry
- $G_{\mu\nu}^{(\text{Obidi})}$= emergent spacetime geometry
- Δ_{Obidi}= the "cost" of converting information geometry into physical spacetime

Thus:

Δ_{Obidi} is the entropic curvature correction that makes Einstein gravity emerge from information geometry.

A.8.1 Worked Example with a Two-Parameter Gaussian Statistical Model of the Obidi Transformation

Model. Consider the family of normal distributions $p(x \mid \mu, \sigma)$with parameters $\theta^1 = \mu$and $\theta^2 = \sigma > 0$.

NB: The Obidi Matricial Notation (TOMN)

In what follows, we have invoked the Obidi Matricial Notation (TOMN), where, in a matrix, a comma denotes a left downward demotion/shift of the immediate set of character(s) that follows:

$$G_{ab} = (G_{00} \quad G_{0i}, G_{i0} \quad G_{ij}) \equiv \begin{pmatrix} G_{00} & G_{0i} \\ G_{i0} & G_{ij} \end{pmatrix}$$

The comma "," is a visual operator that means:

"drop the next component(s) down to the next row."

This is **Obidi's syntactic operator (OSO)**, not a mathematical one. It is Obidi's **notation for layout**, not for algebra.

Obidi Matricial Notation (OMN). In the expressions that follow, we employ the Obidi Matricial Notation, a compact matrix-tensor layout convention introduced for the Theory of Entropicity (ToE). In OMN, a comma "," inside a matrix denotes a **left-downward demotion** of the immediately following entry or block. For example:

$$>G_{ab} >=> \left(> G_{00} \quad G_{0i}, G_{i0} \quad G_{ij} >\right) >\equiv> \begin{pmatrix} > G_{00} & G_{0i} \\ > G_{i0} & G_{ij}> \end{pmatrix}.>$$

This notation provides a clear and economical way to display the natural $0 \boxed{,} - \boxed{,}\, i$ block structure of the **Obidi Metric and its associated transformations**.

Utility of this notation in the Theory of Entropicity (ToE)

The Obidi Metric has a natural block structure:

- the **00 component** (entropy-selected time direction)
- the **0i and i0 components** (mixed components)
- the **ij components** (spatial block)

The OMN notation makes this structure **immediately visible**, which is essential because:

- the Obidi Transformation is rank-one
- the signature flip happens in the 00 direction
- the entropy gradient defines the temporal axis
- the spatial block remains positive definite

Thus, the OMN is not just cosmetic — it is **functionally aligned** with the mathematics of the Theory of Entropicity (ToE).

8.1 Fisher–Rao metric for the Gaussian family

The Fisher information matrix for $\mathcal{N}(\mu, \sigma^2)$(parameters μ, σ) is diagonal:

$$G_{ab}(\mu, \sigma) = (G_{\mu\mu} \quad G_{\mu\sigma}, G_{\sigma\mu} \quad G_{\sigma\sigma}) = \left(\frac{1}{\sigma^2} \quad 0, 0 \quad \frac{2}{\sigma^2}\right). \quad (8.1.1)$$

Its inverse is

$$G^{ab}(\mu, \sigma) = \left(\sigma^2 \quad 0, 0 \quad \frac{\sigma^2}{2}\right). \quad (8.1.2)$$

8.2 Choose the entropy field S

Use the Shannon entropy of the Gaussian (depends only on σ):

$$S(\mu,\sigma) = \frac{1}{2}\ln(2\pi e\,\sigma^2) = \frac{1}{2}\ln(2\pi e) + \ln\,\sigma.\,(8.2.1)$$

Compute partial derivatives:

$$\partial_\mu S = 0, \partial_\sigma S = \frac{1}{\sigma}.\,(8.2.2)$$

8.3 Compute the normalized entropy direction u_a

First compute the squared norm

$$N^2 \equiv G^{ab}\,\partial_a S\,\partial_b S = G^{\sigma\sigma}(\partial_\sigma S)^2 = \frac{\sigma^2}{2}\cdot\frac{1}{\sigma^2} = \frac{1}{2}.\,(8.3.1)$$

Hence $N = 1/\sqrt{2}$. The unit entropy one-form is

$$u_a = \frac{\partial_a S}{N} \Rightarrow u_\mu = 0, u_\sigma = \frac{1/\sigma}{1/\sqrt{2}} = \frac{\sqrt{2}}{\sigma}.\,(8.3.2)$$

Raise indices with G^{ab}:

$$u^\mu = G^{\mu\mu}u_\mu = \sigma^2\cdot 0 = 0, u^\sigma = G^{\sigma\sigma}u_\sigma = \frac{\sigma^2}{2}\cdot\frac{\sqrt{2}}{\sigma} = \frac{\sigma}{\sqrt{2}}.\,(8.3.3)$$

8.4 Apply the Obidi Transformation and Form the Obidi Metric

The Obidi metric is

$$\tilde{G}_{ab} = G_{ab} - 2\,u_a u_b.\,(8.4.1)$$

We now **compute components of the Obidi Metric** (only the $\sigma\sigma$ component is affected because $u_\mu = 0$):

$$\tilde{G}_{\mu\mu} \quad = G_{\mu\mu} - 2u_\mu u_\mu = \frac{1}{\sigma^2} - 0 = \frac{1}{\sigma^2},\ \tilde{G}_{\mu\sigma} \quad = 0 - 2u_\mu u_\sigma = 0,\ \tilde{G}_{\sigma\sigma} \quad = G_{\sigma\sigma} - 2u_\sigma^2.\ (8.4.2)$$

$$\tilde{G}_{\mu\sigma} \quad = 0 - 2u_\mu u_\sigma = 0,\ \tilde{G}_{\sigma\sigma} \quad = G_{\sigma\sigma} - 2u_\sigma^2 = \frac{2}{\sigma^2} - 2\left(\frac{\sqrt{2}}{\sigma}\right)^2 = \frac{2}{\sigma^2} - \frac{4}{\sigma^2} = -\frac{2}{\sigma^2}.\,(8.4.3)$$

So:

$$\boxed{\tilde{G}_{ab}(\mu,\sigma) = \begin{pmatrix}\frac{1}{\sigma^2} & 0, 0 & -\frac{2}{\sigma^2}\end{pmatrix}.\,(8.4.4)}$$

8.5 Checking of Signature for Consistency of the Obidi Transformation

The eigenvalues of $\tilde{G}_{ab}$are the diagonal entries:

$$\lambda_1 = \frac{1}{\sigma^2} > 0, \lambda_2 = -\frac{2}{\sigma^2} < 0. (8.5.1)$$

The determinant is

$$\det \tilde{G} = \frac{1}{\sigma^2} \cdot \left(-\frac{2}{\sigma^2} \right) = -\frac{2}{\sigma^4} < 0. (8.5.2)$$

Conclusion: $\tilde{G}_{ab}$ has one positive and one negative eigenvalue; therefore, it has Lorentzian signature $(-,+)$(one timelike direction and one spacelike direction). **The Obidi Transformation has converted the original Riemannian Fisher–Rao metric into a Lorentzian metric in this simple two-parameter example.**

8.6 Numerical illustration of the Obidi Transformation

Set $\sigma = 1$. Then:

$$G_{ab} = (1 \quad 0, 0 \quad 2), \tilde{G}_{ab} = (1 \quad 0, 0 \quad -2), (8.6.1)$$

clearly showing one positive and one negative eigenvalue.

A.9.1 Theorem (Global signature emergence from the Obidi Transformation)

Statement. Let $(\mathcal{M}_{\text{info}}, G)$ be a connected, smooth n-dimensional manifold equipped with a smooth Riemannian metric G_{ab}(Fisher–Rao). Let $S \in C^\infty(\mathcal{M}_{\text{info}})$be a smooth scalar field (the entropy field). Define the Obidi metric

$$\tilde{G}_{ab} = G_{ab} - 2\, u_a u_b, u_a = \frac{\nabla_a S}{\sqrt{G^{cd} \nabla_c S\, \nabla_d S}}. (9.1.1)$$

Assume the following hypotheses hold globally on $\mathcal{M}_{\text{info}}$:

1. **Nondegenerate gradient (No equilibrium zeros).** $\nabla_a S(\theta) \neq 0$ for every $\theta \in \mathcal{M}_{\text{info}}$.
2. **Uniform transversality / bounded norm.** There exist constants $0 < c_1 \leq c_2 < \infty$ such that

   $$c_1 \leq G^{ab} \nabla_a S\, \nabla_b S \leq c_2 \text{ everywhere on } \mathcal{M}_{\text{info}}.$$

3. **Foliation integrability.** The 1-form $\nabla_a S$ is everywhere hypersurface-orthogonal (i.e., $\nabla_{[a} S\, \nabla_{b]} \nabla_c S = 0$or equivalently $\nabla_a S \wedge d(\nabla S) = 0$), so level sets $\Sigma_t = \{S = t\}$define a smooth codimension-one foliation of $\mathcal{M}_{\text{info}}$.
4. **Topological admissibility.** The normal bundle of the foliation is trivial (so a global unit normal u^aexists) and $\mathcal{M}_{\text{info}}$ admits no topological obstruction to a global Lorentzian metric of signature $(-,+,\ldots,+)$.

Then $\tilde{G}_{ab}$ is a smooth, nondegenerate Lorentzian metric on $\mathcal{M}_{\text{info}}$ with signature $(-,+,\ldots,+)$. Moreover, the level function $t \equiv S$ is a global time function for $(\mathcal{M}_{\text{info}}, \tilde{G})$: its gradient is everywhere timelike with respect to $\tilde{G}$, and the foliation $\{\Sigma_t\}$ is a global spacelike foliation.

Significance of the above Hypotheses

- **(1)** excludes points where the Obidi construction is undefined (division by zero).
- **(2)** prevents degeneracy and controls the normalization u_a; it also gives uniformity needed for global estimates.
- **(3)** ensures the level sets of S are smooth hypersurfaces (no twisting that would prevent a global time foliation).
- **(4)** rules out global topological obstructions (e.g., nonorientable normal bundle) that would forbid a global timelike vector field.

Proof (Prelim)

1. **Well-defined unit field.** By (1) and (2), define

$$u_a = \frac{\nabla_a S}{N}, N \equiv \sqrt{G^{cd}\nabla_c S\, \nabla_d S}, (9.1.2)$$

with $0 < \sqrt{c_1} \le N \le \sqrt{c_2} < \infty$. Thus u_a is smooth and nowhere vanishing.

2. **Algebraic signature check.** For any vector v^a decompose $v^a = v_{\parallel}^a + v_{\perp}^a$ with $v_{\parallel}^a = (u_b v^b)u^a$ and $u_a v_{\perp}^a = 0$. Compute

$$\tilde{G}_{ab} v^a v^b = G_{ab} v^a v^b - 2(u_a v^a)^2 = (u_a v^a)^2(1-2) + G_{ab} v_{\perp}^a v_{\perp}^b. (9.1.3)$$

Since $G_{ab} v_{\perp}^a v_{\perp}^b > 0$ for $v_{\perp} \neq 0$ and the coefficient of $(u_a v^a)^2$ is -1, the quadratic form has one negative direction (along u^a) and $n-1$ positive directions. Hence $\tilde{G}$ has Lorentzian signature $(-,+,\ldots,+)$ pointwise.

3. **Nondegeneracy and smoothness.** The determinant of $\tilde{G}$ satisfies $\det \tilde{G} = -\det G$ for the rank-one update used here (one can check this directly), so $\tilde{G}$ is nondegenerate and smooth because G and u are smooth and u never vanishes.

4. **Timelike gradient and global time function.** Compute $\tilde{G}^{ab}\nabla_a S\, \nabla_b S$. Using $u_a = \nabla_a S/N$ one finds

$$\tilde{G}^{ab}\nabla_a S\, \nabla_b S = G^{ab}\nabla_a S\, \nabla_b S - 2\, \frac{(G^{ab}\nabla_a S\, \nabla_b S)^2}{G^{cd}\nabla_c S\, \nabla_d S} = -\, G^{ab}\nabla_a S\, \nabla_b S < 0, (9.1.4)$$

so $\nabla_a S$ is everywhere timelike for $\tilde{G}$. Thus S is a global time function and level sets Σ_t are spacelike hypersurfaces.

5. **Foliation and global structure.** Hypothesis (3) guarantees the level sets are smooth hypersurfaces; (4) ensures global existence of a continuous choice of unit normal u^a. Together these give a global spacelike foliation with Sas time coordinate.

6. **Conclusion.** Combining the algebraic signature, nondegeneracy, and global foliation yields that $\tilde{G}$is a smooth Lorentzian metric globally on $\mathcal{M}_{\text{info}}$and $S$is a global time function.

Corollaries and stability

- **Local version.** If hypotheses (1)–(3) hold only on an open set $U \subset \mathcal{M}_{\text{info}}$, then $\tilde{G}$is Lorentzian on U. This gives a patchwise (atlas) construction when global hypotheses fail.
- **Stability under small perturbations.** If $S$and $G$are perturbed by sufficiently small C^2changes that preserve (1)–(2) bounds, the conclusion remains valid; the Lorentzian character is C^0-stable because eigenvalues vary continuously.

Obstructions and necessary caveats

- **Zeros of ∇S.** Points where $\nabla S = 0$ are singular for the Obidi map; signature may change or be undefined there. Any global theorem must exclude such critical points or treat them separately (signature-change loci).
- **Topological obstructions.** Some manifolds do not admit a global Lorentzian metric (e.g., certain compact manifolds without boundary). Hypothesis (4) is essential.
- **Nonintegrable entropy 1-form.** If $\nabla_a S$ is not hypersurface-orthogonal, level sets may not be smooth hypersurfaces; one can still obtain a Lorentzian metric but Smay fail to be a global time function.

We may now state all of the above as a formal theorem on the Obidi Transformation of the Obidi Metric:

Theorem. Let $(\mathcal{M}, G)$ be a connected smooth n-manifold with Riemannian metric G. Let $S \in C^\infty(\mathcal{M})$satisfy (i) $\nabla S \neq 0$ on $\mathcal{M}$; (ii) $0 < c_1 \leq G^{ab}\nabla_a S \nabla_b S \leq c_2 < \infty$; (iii) ∇S is hypersurface-orthogonal; (iv) the normal bundle of the foliation $\{S = \text{const}\}$is trivial. Then the Obidi metric $\tilde{G} = G - 2u \otimes u$ is a smooth Lorentzian metric on $\mathcal{M}$and S is a global time function for $(\mathcal{M}, \tilde{G})$.

A.10.1 Scholium on the Obidi Transformation and the Obidi Metric

The Obidi Transformation is what *produces* the Obidi Metric in ToE.

This is the operational relationship:

- The **Obidi Transformation** is the *operation*.
- The **Obidi Metric** is the *result* of applying that operation to the Fisher–Rao information metric.

In other words:

$$\text{Obidi Transformation} \Longrightarrow \text{Obidi Metric} \quad (10.1.1)$$

The transformation is the *cause*. The metric is the *effect*.

A.10.2 Procedure for the Implementation of the Obidi Transformation:

1. Start with the Fisher–Rao information metric

This metric is:

- positive definite
- Riemannian
- purely statistical
- incapable of supporting causality or Lorentzian structure

This is the raw information geometry of the Theory of Entropicity (ToE).

2. Apply the Obidi Transformation

The transformation uses the entropy gradient to:

- select a preferred direction
- flip exactly one eigenvalue
- introduce causal structure
- convert the manifold from Riemannian to Lorentzian

This is the **signature-changing deformation of the Obidi Transformation**.

3. The result is the Obidi Metric

After the transformation, we obtain:

- a Lorentzian metric (of indefinite signature)
- with one timelike direction
- three spacelike directions
- a causal cone structure

- the correct signature for Einstein spacetime

This is the emergent physical metric of ToE.

Thus, the relationship is exactly this:

$$G_{\mu\nu}^{(\text{Obidi})} = \text{ObidiTransform}(g_{\mu\nu}^{(\text{Fisher–Rao})}, \nabla_\mu S) \ (10.1.2)$$

The Obidi Metric **does not exist independently**. It is **generated** by the **Obidi Transformation**.

Comparison Table of the Obidi Transformation and Existing Literature

Feature	**Obidi Transformation [Theory of Entropicity (ToE)]**	**Disformal / rank-one / signature-change literature**
Algebraic form	$\tilde{G} = G - 2\,\frac{\nabla S \otimes \nabla S}{G^{-1}(\nabla S, \nabla S)}$	Disformal: $\tilde{g} = A(\phi)g + B(\phi)\,\partial\phi \otimes \partial\phi$(similar structure).
Physical driver	Entropy gradient ∇S(ontological)	Scalar fields, Killing fields, or embedding directions (various interpretations).
Signature mechanism	Rank-one sign flip along u^a(explicit)	Signature change studied via degeneracy or eigenvalue crossing; transverse models exist.
Global theorem focus	Hypotheses for global emergence (nonzero gradient, foliation, topology)	Literature treats local models, embeddings, and global obstructions separately.
Originality	**Synthesis + physical postulate (entropy as time)**	**Mathematical antecedents present; physical interpretation differs.**

Evidence and nuances

- **Mathematical antecedents exist.** The idea of metrics changing signature, and constructions that produce Lorentzian regions from Riemannian data, are well studied (Kossowski-type models, embedding approaches).

  Signature-changing geometries and constructions that generate Lorentzian regions from Riemannian data have been extensively studied in mathematical relativity, particularly in the work of Kossowski and Kriele on smooth and discontinuous signature transitions [Kossowski–Kriele 1993; Kossowski 1993], as well as in embedding-based approaches such as the Campbell–Magaard theorem [Campbell 1926; Magaard 1963].

- **Disformal and rank-one updates are close relatives.** Bekenstein-style disformal maps and rank-one metric perturbations using a scalar's gradient are standard tools in modified gravity and Kaluza–Klein contexts; algebraically this Obidi transformation is in that family.
- **Novelty of the Obidi Transformation.** The **physical postulate** (entropy field as ontological, gradient as preferred time) plus the **global theorem program** (explicit hypotheses for global signature emergence and pullback to spacetime) are not mere restatements of existing literature; they are a targeted, testable research innovation that goes beyond prior papers' aims in current literature.

Earlier Research Works

We acknowledge that several researchers have built rigorous links from information geometry or entropy to gravitational dynamics, and some derive Einstein-type equations from Fisher/Bures/Fubini–Study structures; however, none of the cited works implements exactly the Obidi prescription (the entropy-gradient, rank-one sign-flip formula plus the global emergence theorems earlier stated).

Key prior results

- **Entropic/Information → Geometry (Caticha).** Caticha develops *Entropic Dynamics* and models space (and attempts spacetime) as a statistical manifold whose metric is Fisher-Rao; he shows strong structural parallels with geometrodynamics but emphasizes the Riemannian (positive-definite) origin and the remaining open problem of producing Lorentz signature.
- **Fisher metric → Einstein tensor (Matsueda et al.).** Matsueda and collaborators derive Einstein-type tensors from Fisher information metrics built from quantum/entanglement data and argue that coarse-graining of information can reproduce energy–momentum content. This is a direct antecedent to "gravity from information."

  Matsueda and collaborators have shown that Fisher information metrics constructed from quantum or entanglement data naturally give rise to Einstein-type curvature tensors, and that coarse-graining of information reproduces effective energy–momentum content [Matsueda 2013; Matsueda 2012; Matsueda–Ishihara–Hashizume 2013]. These works are direct antecedents of modern "gravity from information" programs.
- **Thermodynamic derivations of Einstein field equations (Jacobson).** Jacobson's derivation of Einstein's equation from local thermodynamic relations ($\delta Q = T\, dS$) shows a

deep link between entropy and gravitational dynamics, but it does not proceed by converting a Fisher metric into a Lorentzian metric via a rank-one flip.

- **Recent entropic-action programs.** Recent work (e.g., Bianconi) formulates entropic actions whose IR limit reproduces Einstein-Hilbert–like terms; these are modern, independent realizations of “gravity from entropy.” Although Bianconi’s Gravity-from-Entropy program derives Einstein–Hilbert–like terms from Araki quantum relative entropy, it does not employ disformal deformations, rank-one metric modifications, or entropy-gradient–selected timelike directions.
- **The statistical gravity and entropy of spacetime of Riccardo Fantoni:** Fantoni’s “statistical gravity and entropy of spacetime” program develops a thermal path-integral formulation in which the metric tensor itself is treated as a fluctuating statistical object. As he writes, “we think the metric tensor itself to be in thermodynamic equilibrium at a given temperature” and derives a Euclidean density matrix proportional to exp $\{-T\int(2\kappa R + \mathcal{L}_F)\sqrt{\ ^3g}\, d^3x\}$. This framework does not employ Fisher–Rao information geometry, does not invoke Čencov’s theorem, and does not perform any disformal or rank-one transformation analogous to the **Obidi Transformation**. Consequently, Fantoni’s approach is conceptually adjacent but mathematically distinct from the Theory of Entropicity, which derives Lorentzian spacetime from information geometry via the **Obidi–Čencov Signature Transition Principle.** [Fantoni, Riccardo. 2025. "Statistical Gravity and Entropy of Spacetime" (2025) *Stats* 8, no. 1: 23. https://doi.org/10.3390/stats8010023]
- **Disformal / gradient-dependent metric maps (Bekenstein).** The algebraic pattern of deforming one metric by a scalar gradient (disformal maps) is classical in modified-gravity literature; algebraically this family **overlaps with the Obidi rank-one form but without the specific entropy-as-time postulate of the Theory of Entropicity (ToE).**

Direct comparison of the Obidi Transformation with Existing Work

Aspect	Existing work	Obidi [Theory of Entropicity (ToE)]
Starts from Fisher/Bures info metric	Yes (Caticha, Matsueda).	Yes.
Produces Lorentz signature from Riemannian info metric	Not by the same entropy-gradient rank-one flip; signature-change literature treats type change differently.	**Yes** via explicit Obidi rank-one sign flip.

Aspect	Existing work	Obidi [Theory of Entropicity (ToE)]
Derives Einstein action/equations	Jacobson, Matsueda, Bianconi derive Einstein-like results by thermodynamic or Fisher routes.	**Yes** as an IR limit after pullback + coarse-graining (theorem package).
Physical postulate (entropy = time)	Not emphasized as an ontological postulate in antecedents.	**Central** — entropy gradient chosen as canonical timelike vector.

A.11.1 The Obidi–Čencov Signature Transition Principle: Breaking of Čencov's Theorem and the Emergence of Lorentzian Spacetime

The foundational obstacle confronting any attempt to derive physical spacetime from information geometry is encoded in **Čencov's theorem**, the celebrated uniqueness result of classical statistical geometry. Čencov proved that the **Fisher–Rao metric** is the *only* Riemannian metric on the space of probability distributions that remains invariant under Markov morphisms. This invariance, however, comes at a decisive cost: the Fisher–Rao metric is **strictly positive-definite**, and therefore **incapable** of supporting the Lorentzian signature required for relativistic spacetime.

This tension leads directly to the central structural insight of the Theory of Entropicity (ToE):

A purely Fisher–Rao information manifold cannot become physical spacetime unless the invariance conditions of Čencov's theorem are relaxed or broken.

The **Obidi Transformation** is introduced precisely to accomplish this. By selecting the entropy gradient as a canonical direction and applying a rank-one disformal deformation, the transformation **breaks Čencov invariance** in a controlled and physically motivated manner. The resulting **Obidi Metric** is no longer constrained by the positivity of Fisher–Rao geometry; instead, it acquires a **Lorentzian signature**, a **causal structure**, and a **distinguished temporal axis**.

This leads naturally to the following formal statement.

The Obidi–Čencov Signature Transition Principle (Breaking of Čencov Theorem for Lorentzian Emergence)

Let $(\mathcal{M}, g_{\mu\nu}^{\mathrm{FR}})$ be a Fisher–Rao information manifold satisfying Čencov invariance. Then no Lorentzian metric can be obtained from $g_{\mu\nu}^{\mathrm{FR}}$ without violating the invariance conditions of Čencov's theorem.

The Obidi Transformation provides the minimal entropy-driven deformation that breaks Čencov invariance and yields a Lorentzian metric $G_{\mu\nu}^{(\mathrm{Obidi})}$ *suitable for emergent relativistic spacetime.*

The significance of this proposition is twofold. **First**, it establishes that the emergence of spacetime from information geometry is **not** a passive consequence of the Fisher–Rao structure; it requires an **active entropic deformation**. **Second**, it identifies the **Obidi Transformation** as the **unique structural bridge** between the statistical geometry of information and the physical geometry of General Relativity (GR).

In this sense, the Obidi Proposition is not merely a technical observation but a **conceptual pivot**: it marks the point where the Theory of Entropicity (ToE) departs from classical information geometry and enters the domain of **entropic spacetime physics**:

The Fisher–Rao metric supplies the informational substrate; the Obidi Transformation supplies the entropic mechanism; and the resulting Obidi Metric supplies the Lorentzian geometry from which the Einstein Field Equations ultimately emerge.

A.12 The Theory of Entropicity (ToE) Derivation of Einstein Geometry and the Total Entropic Source Tensor (TEST) of General Relativity (GR)

In Section 6 of this ToE *Letter III* , we explicitly identified the decisive unfinished momentous task of the Theory of Entropicity (ToE) program: not merely to entropically reinterpret spacetime curvature, but to derive **both** the geometric tensor on the left-hand side of Einstein's equations and the source tensor on the right-hand side from a single entropic-information substrate. We have also stated, unequivocally, that pressure, radiation, momentum flow, and stress must be understood as tensorial embodiments of organized entropy rather than as primitives external to the theory. In Section 9, given those insights of Section 6, we thereupon embarked on the arduous task of clarifying our audacious vision with mathematical rigor and precision.

This Section of the Appendix is therefore written to add to [and complete, at least for the time being] that program which we began in Section 9 in a mathematically explicit way.

Our singular strategy here is to introduce [or re-introduce with further nuances] two precise mathematical devices that are not yet isolated sharply enough in the current exposition. The first is an **Obidi Lorentzian Lift (OLL)**, which turns positive-definite information geometry into Lorentzian spacetime geometry by selecting the entropy gradient as the preferred causal direction. The second is an **Entropic Moment Map (EMM)**, which turns localized information distributions into a **symmetric rank-two tensor** whose **perfect-fluid**, **dust**, **radiation**, and **anisotropic-stress limits (ASL)** reproduce the **Einstein stress-energy tensor (ESET)**.

These two devices make the ToE claim exact in form:

Entropy appears on the left as information curvature after Lorentzian lifting, and on the right as localized, transported, and constrained information after moment-taking and metric variation.

Earlier information-geometric and entropic-gravity programs already show parts of this pathway, including information-geometric reconstructions of spacetime, Fisher-metric derivations of Einstein-type tensors, and entropic actions reducing to Einstein gravity in low-coupling limits; the present appendix is designed to present to the reader the unique trajectory undertaken by ToE to derive *both sides [LHR and RHS of the Einstein Field Equations] together* from one parent framework.

12.1 Core operators and foundational definitions

Let $(\mathcal{M}_N)$ be an (N)-dimensional statistical manifold coordinatized by (Θ^A), with $(A, B, \dots = 1, \dots, N)$, and let $(p(y \mid \Theta))$ be a smooth family of probability densities. The Fisher information metric is

$$I_{AB}(\Theta) = \int d\,\mu(y), p(y \mid \Theta), \partial_A \ln p\,(y \mid \Theta), \partial_B \ln p\,(y \mid \Theta). \qquad (A.1)$$

The associated Amari–Čencov cubic tensor is

$$C_{ABC}(\Theta) = \int d\,\mu(y), p(y \mid \Theta), \partial_A \ln p\,, \partial_B \ln p\,, \partial_C \ln p\,, \qquad (A.2)$$

and, with the Levi–Civita connection of (I_{AB}) denoted by $(\Gamma^{(0)A}{}_{BC})$, one may write the (α)-connection family as

$$\Gamma^{(\alpha)A}{}_{BC} = \Gamma^{(0)A}{}_{BC} \frac{\alpha}{2} I^{AD} C_{DBC}. \qquad (A.3)$$

The relevance of these structures is standard in information geometry: the Fisher metric is the unique symmetric two-tensor, up to scale, characterized by invariance under sufficient statistics, and the Amari–Čencov tensor is the canonical cubic companion to that geometry. Precisely because the Fisher metric is positive definite, it cannot by itself produce a Lorentzian causal structure. That obstruction is the mathematical reason ToE needs a controlled deformation beyond untouched Čencov invariance.

Introduce now the Theory of Entropicity (ToE) entropy field $\big(S(\Theta)\big)$. Further define:

$$Q(\Theta) = I^{AB} \partial_A S \partial_B S, \qquad n_A = \frac{\partial_A S}{\sqrt{Q}}, \qquad I^{AB} n_A n_B = 1. \qquad (A.4)$$

The **Obidi Lorentzian Lift** is then the map:

$$\mathcal{L}_S[I]AB := \hat{G}AB\Omega^2(\Theta), (I_{AB} - \sigma, n_A n_B), \qquad \sigma > 1. \qquad (A.5)$$

This is a gradient-dependent **disformal deformation** in the sense of **Bekenstein**, but here its distinguished direction is not an arbitrary scalar field: it is the Theory of Entropicity (ToE) entropy gradient itself. The condition $(\sigma > 1)$ is the minimal condition ensuring one and only one timelike direction.

A short proof is immediate as follows:

Any vector (v^A) decomposes uniquely as $(v^A = v_\perp^A + \lambda n^A)$, with $(n_A v_\perp^A = 0)$. Then

$$\hat{G}_{AB} = v^A v^B \Omega^2 (I_{AB} v_\perp^A v_\perp^B + (1 - \sigma)\lambda^2). \qquad (A.6)$$

Because (I_{AB}) is positive definite and $(1-\sigma<0)$, the quadratic form has exactly one negative eigendirection and $(N-1)$ positive eigendirections. Hence $(\hat{G}_{AB})$ has Lorentzian signature $\big((-+\cdots+)\big)$. And by the matrix-determinant lemma, we therefore have:

$$\det\hat{G}\,\Omega^{2N}(1-\sigma)\det I\,, \qquad (A.7)$$

so that the sign flip is explicit at the level of the determinant as well. **This is the precise mathematical step by which the Theory of Entropicity (ToE) converts statistical/probabilistic [information] distinguishability into causal [physical] geometry.**

Let $(X:,M_4\hookrightarrow\mathcal{M}_N)$ be a smooth coarse-grained four-dimensional section, with spacetime coordinates (x^μ), $(\mu,\nu=0,1,2,3)$. The emergent spacetime metric is thus the pullback:

$$g_{\mu\nu}(x)=\partial_\mu X^A\partial_\nu X^B\hat{G}_{AB}\big(X(x)\big). \qquad (A.8)$$

This is the exact place where physical spacetime appears in the construction logic of the Theory of Entropicity (ToE). The move from information manifold to spacetime by pullback is structurally consonant with earlier information-geometric constructions of **geometrodynamics**, but **ToE's distinct move is to insert the entropy-selected Lorentzian lift *before* the reduction, not after it.**

The second new operator introduced by the Theory of Entropicity (ToE) is the **Entropic Moment Map (EMM)**:

Let $(f_q(x,p))$ be a nonnegative local information distribution on the future mass shell $(\mathcal{P}_x\subset T_xM_4)$, written in ToE's non-extensive sector as:

$$f_q(x,p)=Z_q^{-1}(x),\exp_q!\left[-\alpha(x)-\beta_\mu(x)p^\mu\right], \qquad \exp_q(z):=[1+(1-q)z]^{1/(1-q)}. \qquad (A.9)$$

The invariant mass-shell measure may therefore be taken as

$$dP=\frac{d^4p}{(2\pi)^3}\delta!\left(g_{\mu\nu}p^\mu p^\nu+m^2c^2\right)\Theta(p^0). \qquad (A.10)$$

Then define, for each $(n\geq 0)$,

$$\mathfrak{M}_n\left[f_q\right]^{\mu_1\cdots\mu_n}:=\int_{\mathcal{P}_x}p^{\mu_1}\cdots p^{\mu_n}f_q(x,p)dP. \qquad (A.11)$$

The zeroth, first, and second moments are respectively the local information density, flow, and source tensor; in particular:

$$N_{\text{ent}}^{\mu} = \mathfrak{M}_1[f_q]^{\mu} = \int d\,P p^{\mu} f_q, \qquad \Theta_{\text{ent}}^{\mu\nu} = \mathfrak{M}_2[f_q]^{\mu\nu} = \int d\,P p^{\mu} p^{\nu} f_q. \qquad (A.12)$$

That is, the entropic information current is the first covariant moment of the informational distribution; and the entropic stress–energy tensor is the second covariant moment of the informational distribution f_q.

This second moment is the mathematically exact answer to the question posed earlier in Section 6 of this *Letter III* of the Theory of Entropicity (ToE): **a scalar or probability density does not go directly onto the right-hand side of Einstein's field equations of GR**; rather, its **second covariant moment** does.
The above Equations (A.12) constitute a **fiber integral** over the momentum fiber at each spacetime point.
Theory of Entropicity (ToE) thus applies the fiber structure to entropic **information, such that**:
1. The **entropic information density** is like f.
2. The **Obidi fiber integrals** produce the second moment.
3. The result is a **rank-two entropic source tensor (EST)**, from which the total entropic stress tensor (TEST) is constructed.

Mathematically, we know that is how localized information becomes a rank-two source tensor as fiber integrals. In relativistic kinetic theory, these **fiber integrals** define the **particle current** and the **stress-energy tensor** (**SET**) and, when the kinetic equation holds with **collision invariants**, they are **divergence-free** in exactly the sense required by the **Bianchi identity**.

In the Theory of Entropicity (ToE), information does not enter Einstein's equations as a scalar density but through its second covariant moment. This second moment is obtained by fiber integrals over the information distribution, exactly as in relativistic kinetic theory where the stress–energy tensor is the second moment of the particle distribution. These fiber integrals convert localized information into a rank-two source tensor representing mass density, momentum flux, pressure, stress, and radiation. When the underlying kinetic equation satisfies

collision invariants, the resulting entropic stress–energy tensor is automatically divergence-free, matching the Bianchi identity and making it the correct RHS of the Einstein field equations.

In the Theory of Entropicity (ToE), information is the fundamental microscopic quantity. Mass, energy, momentum, pressure, stress, and radiation are macroscopic expressions of coarse-grained informational structure. The scalar information density does not directly source curvature; instead, its second covariant moment — obtained through informational fiber integrals — produces a symmetric, divergence-free rank-two tensor that plays the role of the stress–energy tensor in Einstein's equations.

Mass, energy, momentum, pressure, stress, and radiation all *contain* information, and their macroscopic forms arise from coarse-graining underlying informational degrees of freedom.

Thus, the Theory of Entropicity (ToE) is asserting the following structural law:

Localized information becomes a physical source only through its second covariant moment, obtained by fiber integrals over the information distribution.

That is, the Theory of Entropicity (ToE) is teaching us that information becomes mass, momentum, pressure, stress, and radiation only after taking its second covariant moment via fiber integrals — exactly as particles do in kinetic theory.

12.2 Theorem (Entropic Origin of the Einstein Stress–Energy Tensor)

Theorem (Obidi Entropic Stress–Energy Theorem).

We must now summarize much of what we have presented in the foregoing with a theorem.

Let $(\mathcal{M}, g_{\mu\nu})$be a Lorentzian spacetime obtained from an underlying Fisher–Rao information manifold via the Obidi–Čencov Signature Transition Principle. Let $f_q(x,p)$be a non-negative informational distribution function on the mass shell over $\mathcal{M}$, and let

$$dP = \frac{d^4p}{(2\pi)^3}\,\delta(g_{\mu\nu}p^\mu p^\nu + m^2c^2)\,\Theta(p^0) \quad (A.12.1)$$

be the invariant momentum-space measure. Define the **entropic stress–energy tensor** by the second covariant moment

$$\Theta^{\mu\nu}_{\text{ent}}(x) \;=\; \int p^{\mu}p^{\nu} f_q(x,p)\, dP. \text{(A. 12.2)}$$

Assume that f_q satisfies an informational kinetic equation of **Boltzmann/Vlasov** type with **collision invariants** corresponding to **conservation of informational "charge" and informational four-momentum.** Then:

1. **Tensoriality and symmetry:** $\Theta^{\mu\nu}_{\text{ent}}$ is a symmetric rank-two tensor field on $\mathcal{M}$.
2. **Covariant conservation:**

$$\nabla_{\mu}\Theta^{\mu\nu}_{\text{ent}} = 0, \text{(A. 12.3)}$$

i.e. the entropic stress–energy tensor is divergence-free.

3. **Einstein coupling:** There exists a constant κ_{ent} such that the Einstein field equations

$$G_{\mu\nu} = \kappa_{\text{ent}}\, \Theta^{\mu\nu}_{\text{ent}} \text{ (A. 12.4)}$$

are consistent with the **Bianchi identity** $\nabla_{\mu}G^{\mu\nu} = 0$ and the **informational kinetic dynamics** of f_q.

In particular, **the scalar informational distribution f_q does not enter Einstein's equations directly**; rather, its **second covariant moment** $\Theta^{\mu\nu}_{\text{ent}}$ is the unique divergence-free rank-two tensor that can serve as the **macroscopic gravitational source**. Thus, **in the Theory of Entropicity (ToE), mass, momentum, pressure, stress, and radiation are coarse-grained manifestations of underlying informational structure encoded in f_q**, and **the Einstein stress–energy tensor is recovered as the entropic second moment of information.**

12.3 Derivation of the Geometric Side (LHS) of Einstein's Field Equations in ToE

The Theory of Entropicity (ToE) **parent information-gravity action** (**PIGA**) [which essentially is another formulation of the **Obidi Action**] is taken in (N) dimensions to be

$$A_{\text{IG}} = \frac{1}{2\kappa_I}\int_{\mathcal{M}_N} d^N\,\Theta, \sqrt{|\hat{G}|}\left(\mathcal{R}[\hat{G}] - 2\Lambda_I\right) + A^{(N)}_{\text{src}}[\hat{G}, S, f_q, \zeta_I], \qquad (A.13)$$

where (κ_I) is the information-gravitational stiffness, (Λ_I) is the parent vacuum term, and (ζ_I) denotes whatever additional internal information variables are needed to describe localization, binding, or structural constraints.

Assume that, on the coarse-grained sector, the lifted information metric admits a block form

$$\hat{G}_{AB} = d\Theta^A d\Theta^B g_{\mu\nu}(x), dx^\mu dx^\nu + h_{ab}(x,y), \left(dy^a + A_\mu^a dx^\mu\right)\left(dy^b + A_\nu^b dx^\nu\right), \qquad (A.14)$$

with $(a, b = 1, \dots, N-4)$.

Then the measure factorizes as:

$$\sqrt{|\hat{G}|} = \sqrt{-g}\sqrt{h}, \qquad h := \det(h_{ab}), \qquad (A.15)$$

and, after integrating over the internal information fibre (F), one gets the four-dimensional effective action:

$$A_{\text{eff}} = \frac{1}{2\kappa_{\text{eff}}}\int_{M_4} d^4x\sqrt{-g}(R[g] - 2\Lambda_{\text{ent}}) + A_{\text{src}}^{(4)} + A_{\text{corr}}. \qquad (A.16)$$

Here:

$$\kappa_{\text{eff}}^{-1} = \kappa_I^{-1}\int_F d^{N-4}y\sqrt{h}, \qquad \Lambda_{\text{ent}} = \Lambda_I \frac{1}{2}\langle\mathcal{R}_{\text{int}} + \mathcal{R}_{\text{mix}}\rangle_F, \qquad (A.17)$$

where $(\mathcal{R}_{\text{int}})$ is the internal information-fibre curvature and $(\mathcal{R}_{\text{mix}})$ is the fibre/base mixing curvature. The correction action (A_{corr}) collects heavy information modes, higher-derivative terms, and non-adiabatic residues. At this point the Einstein–Hilbert sector is not postulated; it has been isolated as the infrared part of the lifted information action. **This is exactly the sort of low-coupling Einsteinian limit that already appears in neighboring entropic-gravity constructions, but here its origin is the Lorentzian lift of information geometry plus fibre reduction.**

Now vary (A_{eff}) with respect to $(g^{\mu\nu})$. Using the Palatini identity in the standard form, we readily obtain:

$$\delta\left(\sqrt{-g}, R\right) = \sqrt{-g}, \left(G_{\mu\nu}, \delta g^{\mu\nu} + \nabla_\alpha V^\alpha\right), \qquad (A.18)$$

and discarding the total divergence by the usual boundary prescription, one arrives at:

$$\delta A_{\text{eff}} = \frac{1}{2}\int d^4x, \sqrt{-g}, \left[\frac{1}{\kappa_{\text{eff}}}\left(G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu}\right) T_{\mu\nu}^{\text{ToE}} \Delta_{\mu\nu}^{\text{IG}}\right] \delta g^{\mu\nu}, \qquad (A.19)$$

where:

$$T_{\mu\nu}^{\text{ToE}} = -\frac{2}{\sqrt{-g}}\frac{\delta A_{\text{src}}^{(4)}}{\delta g^{\mu\nu}}, \qquad \Delta_{\mu\nu}^{\text{IG}} = -\frac{2}{\sqrt{-g}}\frac{\delta A_{\text{corr}}}{\delta g^{\mu\nu}}. \qquad (A.20)$$

Hence the ToE field equations are

$$G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu} = \kappa_{\text{eff}} T_{\mu\nu}^{\text{ToE}} + \kappa_{\text{eff}} \Delta_{\mu\nu}^{\text{IG}}. \qquad (A.21)$$

If one writes ($\kappa_{\text{eff}} = 8\pi G_{\text{eff}}/c^4$), the left-hand side of Einstein's equations has now been generated explicitly from information geometry:

$$G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu} = \frac{8\pi G_{\text{eff}}}{c^4}\left(T_{\mu\nu}^{\text{ToE}} + \Delta_{\mu\nu}^{\text{IG}}\right). \qquad (A.22)$$

The geometric side is therefore the coarse-grained curvature sector of the Lorentz-lifted information manifold. No separate metric postulate is needed at the effective level beyond the existence of the section ($X(M_4)$).

12.4 Derivation of the source side (RHS) of Einstein's Field Equations in ToE

The right-hand side (RHS) of the Einstein Field Equations is where ToE must be strongest. The source tensor cannot be a scalar entropy placed by hand on the RHS; it must emerge as the tensorial embodiment of localized information. The proper four-dimensional source action is therefore decomposed as follows:

$$A_{\text{src}}^{(4)} = A_S + A_{\text{kin}} + A_{\text{cons}}, \qquad (A.23)$$

corresponding respectively to the **coherent entropy-field sector**, the **incoherent or kinetic-distribution sector**, and **the localization/constraint sector**.

The coherent sector is:

$$A_S = \int d^4\, x, \sqrt{-g}, [K(X,S) - U(S)], \qquad X := -\frac{1}{2}\nabla_\mu S \nabla^\mu S. \qquad (A.24)$$

Its Hilbert stress tensor is:

$$T_{\mu\nu}^{(S)} = K_X, \nabla_\mu S \nabla_\nu S + g_{\mu\nu}[K(X,S) - U(S)]. \qquad (A.25)$$

If $(\nabla_\mu S)$ is timelike, define:

$$u_\mu = \frac{\nabla_\mu S}{\sqrt{2X}}, \qquad u_\mu u^\mu = -1, \qquad h_{\mu\nu} := g_{\mu\nu} + u_\mu u_\nu. \qquad (A.26)$$

Then $(T^{(S)}_{\mu\nu})$ assumes the exact perfect-fluid form:

$$T^{(S)}_{\mu\nu} = \rho_S, u_\mu u_\nu + p_S, h_{\mu\nu}, \qquad \rho_S = 2XK_X - K + U, \qquad p_S = K - U. \qquad (A.27)$$

For the canonical choice $(K = X)$, this becomes:

$$T^{(S)}_{\mu\nu} = \nabla_\mu S \nabla_\nu S g_{\mu\nu} \left[\frac{1}{2}\nabla_\alpha S \nabla^\alpha S + U(S)\right]. \qquad (A.28)$$

This is important for ToE because it proves that even a single coherent entropic mode already generates a legitimate covariant source tensor, and that when its gradient is timelike it is indistinguishable from an effective fluid. That is exactly the classical scalar-fluid correspondence known in GR and k-essence [theory].

The incoherent, particulate, radiative, or coarse-grained sector is encoded by the **Entropic Moment Map (EMM)** introduced earlier above. Its fundamental source tensor is

$$\Theta^{\text{ent}}_{\mu\nu} = \int_{\mathcal{P}_x} d\,P, p_\mu p_\nu, f_q(x,p). \qquad (A.29)$$

Choose the Landau velocity (u^μ) by

$$\Theta^{\mu\nu}\text{ent} = u\nu - \rho_{\text{kin}} u^\mu, \qquad u_\mu u^\mu = -1. \qquad (A.30)$$

Then the exact (1+3) decomposition of the kinetic entropic source is

$$\Theta^{\text{ent}}_{\mu\nu} = \rho_{\text{kin}} u_\mu u_\nu + p_{\text{kin}} h_{\mu\nu} + 2u_{(\mu} q_{\nu)} + \pi_{\mu\nu}, \qquad (A.31)$$

with projected parts:

$$\rho_{\text{kin}} = u^\mu u^\nu \Theta^{\text{ent}}_{\mu\nu}, \qquad p_{\text{kin}} = \frac{1}{3} h^{\mu\nu} \Theta^{\text{ent}}_{\mu\nu}, \qquad (A.32)$$

$$q_\mu = h_\mu{}^\alpha u^\beta \Theta^{\text{ent}}_{\alpha\beta}, \qquad \pi_{\mu\nu} = \left(h_{(\mu}{}^\alpha h_{\nu)}{}^\beta - \frac{1}{3} h_{\mu\nu} h^{\alpha\beta}\right) \Theta^{\text{ent}}_{\alpha\beta}. \qquad (A.33)$$

In the above, $q_\mu$is the **entropic heat (or energy) flux four-vector**: the part of $\Theta^{\text{ent}}_{\alpha\beta}$ that is orthogonal to u^μin one index and contracted with u^βin the other; and $\pi_{\mu\nu}$ is the **symmetric, trace-free, spatial projection** of $\Theta^{\text{ent}}_{\alpha\beta}$.

In the above:

- $u^\beta$is the observer's four-velocity,
- $h_\mu^{\ \alpha} = \delta_\mu^{\ \alpha} + u_\mu u^\alpha$is the spatial projector,
- $\Theta^{\text{ent}}_{\alpha\beta}$is the ToE entropic stress–energy tensor.

This is the mathematically decisive identification for the RHS of Einstein's field equations. In ToE language, we write:

1. mass density; $\equiv$; $\frac{\rho_{\text{kin}}}{c^2}$ in the rest frame,
2. pressure; $\equiv$; $\frac{1}{3} h^{\mu\nu} T_{\mu\nu}$,
3. momentum flux / heat flow; $\equiv$; q_μ,
4. anisotropic stress / shear stress; $\equiv$; $\pi_{\mu\nu}$.

Thus, pressure, radiation, and stress are not verbal appendages added to entropy; they are **different projections of the same rank-two entropic moment tensor (EMT)**. This is the direct mathematical path we have employed to state the intuition already present in this *Letter III*: **localized information becomes physical matter when it acquires inertia, flow, resistance, and anisotropy under coarse-graining.** In relativistic kinetic theory, the tensor in (A.29) is symmetric and conserved whenever the microscopic distribution obeys the **Liouville or Boltzmann equation** with the usual **collision invariants.**

The localization or constraint sector captures bound structure, elastic response, compositional locking, topological storage, and other organizational features of information that are not exhausted by a one-particle distribution.

Next, we invoke:

$$A_{\text{cons}} = \int d^4 x, \sqrt{-g}, \Lambda_c\big(n, s, \zeta_I, B_{IJ}, \dots\big), \qquad (A.34)$$

where (n) is the local carrier density, (s) the entropy per carrier, (ζ_I) internal information labels, and (B_{IJ}) any additional strain or organization invariants. Then, we have:

$$\Sigma_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta A_{\text{cons}}}{\delta g^{\mu\nu}} \qquad (A.35)$$

is the **entropic constraint stress tensor**. This is the term that allows the Theory of Entropicity (ToE) to describe **bound matter**, **internal stresses**, **elastic response**, and **stable macroscopic identity** without pretending that all matter is a free gas. It is the rigorous home for "structured information" in the source sector. **Fluid and dust actions** of this general type are of course standard in relativistic variational hydrodynamics.

The full ToE source tensor is therefore given as:

$$T_{\mu\nu}^{\text{ToE}} = T_{\mu\nu}^{(S)} + \Theta_{\mu\nu}^{\text{ent}} + \Sigma_{\mu\nu}. \qquad (A.36)$$

Equivalently, relative to the hydrodynamic velocity (u^{μ}), we have:

$$T_{\mu\nu}^{\text{ToE}} = \rho_{\text{eff}} u_{\mu} u_{\nu} + p_{\text{eff}} h_{\mu\nu} + 2u_{(\mu} q_{\nu)} + \pi_{\mu\nu}, \qquad (A.37)$$

such that:

$$\rho_{\text{eff}} = u^{\mu} u^{\nu} T_{\mu\nu}^{\text{ToE}}, \qquad p_{\text{eff}} = \frac{1}{3} h^{\mu\nu} T_{\mu\nu}^{\text{ToE}}, \qquad q_{\mu} = -h_{\mu}{}^{\alpha} u^{\beta} T_{\alpha\beta}^{\text{ToE}}, \qquad \pi_{\mu\nu} = T_{\langle\mu\nu\rangle}^{\text{ToE}}. \qquad (A.38)$$

In this form, every familiar Einstein source sector is already present: dust, radiation, perfect fluids, viscous media, anisotropic stresses, coherent scalar media, and bound organizational matter. Thus, once again, we recognize that the ToE claim is not that scalar entropy equals ($T_{\mu\nu}$); it is that **the entropic field, together with its currents, distributions, and constraints, generates the unique rank-two object that coarse-grains to ($T_{\mu\nu}$)**. That is exactly the source-side completion demanded by Section 6 of this *Letter III*.

The conservation law now follows in this way:

If (f_q) obeys the **relativistic Boltzmann equation** (**RBE**),

$$p^{\alpha} \nabla_{\alpha} f_q = C[f_q], \qquad (A.39)$$

and the **collision term** satisfies the **moment invariant**

$$\int d\, P p^{\nu} C[f_q] = 0, \qquad (A.40)$$

then:

$$\nabla_\mu \Theta^{\mu\nu}_{\text{ent}} = 0. \qquad (A.41)$$

If the coherent entropy field satisfies its **Euler–Lagrange equation** and the constraint variables (ζ_I) satisfy their own **variational equations**, then **diffeomorphism invariance** of the full action implies:

$$\nabla_\mu T^{\mu\nu}_{\text{ToE}} = 0. \qquad (A.42)$$

At equilibrium, the same structure can be recovered from a **thermodynamic generating functional**. In **covariant statistical mechanics** (**CSM**), the **equilibrium stress-energy tensor** (**ESET**) can be obtained as a functional derivative of the **partition functional** with respect to **the inverse-temperature four-vector**, and in **relativistic fluid actions** the **equilibrium Lagrangian** may be written in terms of pressure. **This gives us a second, independent route from entropy to source tensor, now from the partition-functional side rather than the moment side.**

Finally, the local Second Law of Thermodynamics has its proper place here. The entropy current (J^μ_{ent}) of the non-equilibrium effective theory satisfies

$$\nabla_\mu J^\mu_{\text{ent}} \geq 0, \qquad (A.43)$$

so that the **entropic source tensor** is not merely conserved; it also carries the **irreversible bookkeeping** required by **ToE's arrow-of-time sector**. **Modern non-equilibrium EFT** derivations show that **local entropy-production positivity** follows from **symmetry** and **unitarity** assumptions in the **hydrodynamic regime**.

12.5 Einstein Limit and the Unification Theorem of the Theory of Entropicity (ToE)

The low-energy Einstein limit is obtained by taking the simultaneous infrared and local-equilibrium limit:

$$q \to 1, \qquad \Delta^{\text{IG}}_{\mu\nu} \to 0, \qquad q_\mu \to 0, \qquad \pi_{\mu\nu} \to 0, \qquad \Sigma_{\mu\nu} \to \Sigma^{\text{eq}}_{\mu\nu}. \qquad (A.44)$$

Then (f_q) reduces to the ordinary **Maxwell–Jüttner/Boltzmann sector**, the **dissipative pieces disappear, the information-geometry corrections decouple, and the full ToE source tensor reduces to the generalized perfect-fluid form:**

$$T^{\text{ToE}}_{\mu\nu} \longrightarrow (\rho + p)u_\mu u_\nu + p, g_{\mu\nu}. \qquad (A.45)$$

This is the Einstein fluid tensor. The special cases are immediate:

$$p \ll \rho \quad \Longrightarrow \quad T_{\mu\nu} \approx \rho, u_\mu u_\nu \qquad \text{(dust / cold matter)}, \qquad (A.46)$$

$$m = 0, \quad \rho = 3p \quad \Longrightarrow \quad T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p, g_{\mu\nu} \qquad \text{(radiation)}, \qquad (A.47)$$

$$K = X \quad \Longrightarrow \quad T_{\mu\nu}^{(S)} = \nabla_\mu S \nabla_\nu S g_{\mu\nu} \left[\frac{1}{2} \nabla_\alpha S \nabla^\alpha S + U(S)\right] \qquad \text{(canonical scalar sector)}. \qquad (A.48)$$

The scalar-field result is not foreign to Einstein matter; it is one of its standard admissible sectors, and when ($\nabla_\mu S$) is timelike it is already equivalent to a perfect fluid. Thus, the RHS of Einstein field equations is recovered from the Theory of Entropicity (ToE) not by forcing entropy into a pre-existing slot, but by passing from the entropic field, through moment formation and variational projection, right into the familiar General Relativity (GR) matter classes.

The final effective field equation of the completed ToE program is therefore this:

$$G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu} \frac{8\pi G_{\text{eff}}}{c^4}, T_{\mu\nu}^{\text{ToE}} + \frac{8\pi G_{\text{eff}}}{c^4}, \Delta_{\mu\nu}^{\text{IG}}. \qquad (A.49)$$

In the strict Einstein limit, we have:

$$G_{\text{eff}} \to G, \qquad \Lambda_{\text{ent}} \to \Lambda, \qquad \Delta_{\mu\nu}^{\text{IG}} \to 0, \qquad T_{\mu\nu}^{\text{ToE}} \to T_{\mu\nu}^{\text{Einstein}}, \qquad (A.50)$$

and the **Theory of Entropicity (ToE) thus recovers the celebrated Einstein Field Equations (EFE) of General Relativity (GR):**

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\text{Einstein}}. \qquad (A.51)$$

In closing, a compact theorem may now be stated as a befitting summary of our efforts so far:

12.6 Theorem: Obidi's Relativistic Reduction Theorem (ORRT)

Let $\big((\mathcal{M}_N, I_{AB}, S)\big)$ be a Theory of Entropicity (ToE) information manifold with nonvanishing entropy gradient on a smooth four-dimensional coarse-grained section ($X(M_4)$). Let the Fisher metric be **Lorentz-lifted** by the **Obidi map** $\big(\hat{G}_{AB} = \mathcal{L}_S[I]AB\big) with (\sigma > 1)$*, let the* ***lifted action*** *reduce to (A.16)* ***after fibre integration****, and let the* ***microscopic information*** *content be encoded by* *the **moment-generating distribution** (f_q) together with **coherent** and **constraint sectors** (A_S) and* (Acons). Then the effective four-dimensional field equation is (A.49). In the **infrared local-equilibrium Boltzmann–Gibbs limit**, it reduces exactly to the **Einstein field equations of General Relativity** (A.51).

The proof is the reasoning chain already established above: the Lorentzian lift generates the causal metric; the pullback and fiber reduction generate the Einstein tensor; the moment map and Hilbert variation generate the source tensor; and the equilibrium limit suppresses non-Einstein corrections. The geometric side and the source side are thus **two tensorial manifestations of the same underlying entropic-information structure (EIS)**, which is precisely the monistic claim of the Theory of Entropicity (ToE), now written in a fully covariant mathematical form. This conclusion fulfills the ambition stated at the outset of *Letter III*, while making explicit the **derivational machinery** developed and justified throughout the preceding Sections.