Letter IB

ToE Living Review Letters IB:

On the Haller-Obidi Action and Lagrangian: *An Examination of the Mathematical and Conceptual Connection Between John Haller's Action-as-Entropy Equivalence and the Entropic Field Obidi Action Formulation of the Theory of Entropicity (ToE)*

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> **"The principle of least action is the most general and the most powerful method known for the formulation of the laws of physics."**
>
> — *Richard P. Feynman*, The Feynman Lectures on Physics *(1964)*
>
> **"The laws of physics must be such that they apply to a world in which information is the fundamental currency."**
>
> — *John Archibald Wheeler*, It from Bit *(1989)*

> **"Entropy is a measure of our ignorance of the microscopic state of the system."**
>
> — *Edwin T. Jaynes,* Information Theory and Statistical Mechanics *(1957)*
>
> **"The gravitational field equations can be viewed as an equation of state, arising from the thermodynamics of spacetime."**
>
> — *Ted Jacobson,* Thermodynamics of Spacetime *(1995)*

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ABSTRACT

This Letter [***Letter IB*** in the **Theory of Entropicity (ToE) Living Review Letters Series**] presents a rigorous mathematical examination of the structural and formal connections between John L. Haller Jr.'s 2015 entropy-action identity — $H = (2/\hbar)\int(mc^2 - L)dt$ — and the Obidi entropic field action formulation of the Theory of Entropicity (ToE). We define the **Haller-Obidi Action** as the explicit single-particle entropic action $S_{HO} = \int \mathcal{L}_{HO}\, dt$ whose Lagrangian $\mathcal{L}_{HO} = mc^2 - (\hbar/2)(dH/dt)$ is constructed by rearranging Haller's central result into a variational form that ToE absorbs as a worldline sector. We demonstrate that this **Haller-Obidi Lagrangian** admits a natural covariant generalization $\mathcal{L}_{ent} = mc^2 - (\hbar/2)(u^\mu\, \partial_\mu\, S)$ when the entropic field $S(x)$ of ToE is restricted to a particle worldline. The formal reduction of the Obidi Action to the Haller-Obidi Action is established through a localization procedure, proving that Haller's identity is the single-particle projection of the universal entropic field dynamics. We further show that Haller's decomposition $H = H_C + I_M$ maps onto the free-plus-interaction decomposition of the entropic Lagrangian, that the mutual information rate $dI_M/dt = (2/\hbar)V$ provides a concrete prototype for entropic coupling constants and information-geometric potentials, and that the Gaussian channel structure underlying Haller's derivation corresponds to the $\alpha = 0$ (Levi-Civita) sector of ToE's entropic α-connection. We explore the bridge to the Vuli-Ndlela Integral through entropy-weighted path selection, and we identify the precise mathematical limits of the Haller-ToE correspondence — including the absence of an entropic field, conserved entropic flux, and intrinsic time asymmetry in Haller's framework. The Haller-Obidi Action and Lagrangian thus serve as a concrete, calculable bridge between information-theoretic particle mechanics and the full entropic field theory of ToE.

EXECUTIVE SUMMARY

- **Haller (2015)** derives $H = (2/\hbar)\int(mc^2 - L)dt$ from first principles in information theory and quantum diffusion, directly identifying entropy with the classical action. The derivation proceeds through a Bernoulli-Gaussian diffusion model, the Hirshman entropy sum, conditional entropy rates, and a Gaussian mutual information channel — each contributing one structural element to the final identity.
- We construct the **Haller-Obidi Lagrangian** $\mathcal{L}_{HO} \equiv mc^2 - (\hbar/2)\dot{H}$ by rearranging Haller's central result, yielding an explicit entropic effective action at the particle level that admits variational treatment. The resulting Haller-Obidi Action $S_{HO} = \int\mathcal{L}_{HO}\, dt$ is shown to be identically equal to the classical action S_{action}, confirming internal consistency while exposing the informational anatomy of the classical Lagrangian.
- The **covariant generalization** $\mathcal{L}_{ent} = mc^2 - (\hbar/2)(u^\mu\, \partial_\mu\, S)$ connects the Haller-Obidi construction to the entropic field $S(x)$ of ToE, with the entropic current $J^\mu{}_S = \rho_S\, u^\mu$ and the continuity condition $\nabla_\mu\, J^\mu{}_S = 0$ emerging naturally. The covariant formulation extends Haller's non-relativistic identity to arbitrary curved spacetimes.
- The **Obidi Action** $S_{Obidi} = \int F(S, \nabla S, g_{\mu\nu})\, d^4x$ reduces to the Haller-Obidi Action upon worldline localization, establishing the **Obidi-Haller Correspondence** as a rigorous mathematical limit: Obidi Action (field level) → Haller-Obidi Action (worldline level) → Classical Action (non-relativistic limit).
- Haller's mutual information rate $dI_M/dt = (2/\hbar)V$ provides a prototype for **information-geometric potentials** and suggests a route from mutual information to an effective entropic metric $g^{(ent)}{}_{\mu\nu} \sim \partial^2 I_M/\partial\theta^\mu\partial\theta^\nu$, realizing the

Fisher-Rao metric as emergent spacetime geometry in the $\alpha = 0$ sector of ToE's entropic α-connection.

- The entropy-weighted path selection implicit in Haller's framework motivates the **Vuli-Ndlela path integral** $Z_{VN} = \int\mathcal{D}[x] \exp\{iS[x]/\hbar + \lambda H[x]\}$, while the limits of the correspondence are clearly delineated: Haller does not construct an entropic field, conserved flux, or intrinsic time asymmetry — structures that emerge only at the full field-theoretic level of the Obidi Action.

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1. Introduction — From Entropy-Action Identity to Entropic Lagrangian Mechanics

The principle of least action and the concept of entropy have been treated as conceptually distinct pillars of physics for over three centuries. The action functional — Hamilton's integral of the Lagrangian along a worldline — governs the trajectories of particles and the dynamics of fields through a variational principle that selects, from the space of all kinematically admissible histories, the unique path that extremizes the action. Entropy, by contrast, enters physics through the second law of thermodynamics and its information-theoretic generalizations: it measures the multiplicity of microstates consistent with a given macrostate, the uncertainty in a probability distribution, the irreversibility of a dynamical process. Action selects trajectories; entropy counts states. Action is reversible; entropy is directional. Action lives in configuration space; entropy lives in probability space. Or so the canonical wisdom has maintained.

This canonical separation was first challenged at the interface of general relativity and quantum field theory. Bekenstein's 1973 derivation of black hole entropy [2], proportional to the horizon area in Planck units, demonstrated that gravitational dynamics encodes information-theoretic content in its geometry. Jacobson's 1995 thermodynamic derivation of the Einstein field equations from the Clausius relation $\delta Q = T\,\delta S$ applied to local Rindler horizons [3] showed that spacetime curvature could be understood as a macroscopic consequence of microscopic entropic dynamics. Verlinde's 2011 entropic gravity program [4] and Padmanabhan's surface-bulk thermodynamic framework [5] extended this insight, arguing that gravitational acceleration itself is an entropic force emerging from the statistical mechanics of horizon degrees of freedom. Frieden's Fisher information approach [6] and Jaynes' maximum entropy formalism [7] attacked the problem from the information-theoretic side, showing that the equations of motion of classical and quantum mechanics could be derived from optimization principles on probability distributions.

Letter IA of this series — *The Entropic Rosetta Stone* [15] — surveyed this historical and conceptual landscape in detail, establishing the position of the Theory of Entropicity (ToE) as the synthesis and extension of these entropy-as-generator programs. The central claim of Letter IA was that the tradition of deriving dynamics from entropy is not merely a collection of independent results but the partial excavation of a single underlying structure: the entropic field $S(x)$ of ToE, whose dynamics generate geometry, fields, and law from a single informational primitive.

Within that survey, the 2015 paper by John L. Haller Jr., "Action as Entropy" [1], occupied a position of special importance. Haller demonstrated, through a self-contained derivation grounded in quantum diffusion, Bernoulli processes, and information-theoretic entropies, that the total self-information of a quantum particle — defined as the sum of conditional entropy and mutual information — equals the time integral of the mass-energy minus the classical Lagrangian, scaled by the quantum of action:

$H = (2/\hbar) \int (mc^2 - L)\, dt$.. (1)

This is the entropy-action identity: a direct mathematical equation between an information-theoretic quantity (the Hirshman entropy of a quantum diffusion process) and a mechanical quantity (the classical action plus a rest-energy baseline). Letter IA discussed the conceptual significance of this result for ToE. The present Letter — Letter IB — has a different and more specific objective.

The objective of this Letter is to examine the precise mathematical and structural connections between Haller's particle-level entropy-action identity and the entropic field action formulation of the Theory of Entropicity. Where Letter IA asked, "What does Haller's result mean for ToE?", Letter IB asks: "**What mathematical structures connect Haller's particle-level identity to the Obidi field-level action, and what new constructions emerge from their synthesis?**"

The answer, as we shall demonstrate, is rich. From Haller's identity (1) and the Obidi Action of ToE:

$$S_{\text{Obidi}} = \int F(S, \nabla S, g_{\mu\nu}) \sqrt{(-g)}\, d^4x \ldots\ldots \quad (2)$$

we construct two named mathematical objects that serve as the connective tissue between the particle-level and field-level formulations:

(i) The **Haller-Obidi Lagrangian**, $\mathcal{L}_{\text{HO}} \equiv mc^2 - (\hbar/2)\dot{H}$, obtained by rearranging Haller's entropy rate identity into a variational object — a Lagrangian in the mechanical sense that can be subjected to the Euler-Lagrange procedure, yielding equations of motion that are simultaneously mechanical and informational.

(ii) The **Haller-Obidi Action**, $S_{\text{HO}} \equiv \int \mathcal{L}_{\text{HO}}\, dt$, the time integral of the Haller-Obidi Lagrangian, which we show is identically equal to the classical action. The classical action *is* the entropic action, rewritten in information-theoretic variables.

Beyond these definitions, we construct the covariant generalization of the Haller-Obidi Lagrangian by embedding the non-relativistic entropy rate into the entropic field $S(x)$ of ToE, yielding a worldline Lagrangian that couples particle motion to the ambient entropic field through the four-velocity contraction $u^\mu \partial_\mu S$. We prove that the Obidi Action (2) reduces to this covariant Haller-Obidi Action upon localization of the entropic field to a single timelike worldline — establishing the **Obidi-Haller Correspondence** as a rigorous mathematical limit, not merely an analogy.

We further demonstrate that Haller's information-theoretic decomposition $H = H_C + I_M$ maps precisely onto the free-plus-interaction decomposition of the entropic Lagrangian; that the mutual information rate $dI_M/dt = (2/\hbar)V$ provides a concrete prototype for entropic coupling constants; that the Gaussian channel structure of Haller's mutual information calculation corresponds to the $\alpha = 0$ (Levi-Civita) sector of ToE's entropic α-connection; and that the entropy-weighted path selection implicit in Haller's framework motivates the Vuli-Ndlela path integral of ToE. We close by stating honestly and precisely where the mathematical correspondence ends — what structures of ToE have no counterpart in Haller's framework, and what structures of Haller's framework do not survive the passage to the full entropic field theory.

The central task of this Letter is therefore to construct the precise mathematical bridge between equations (1) and (2). The construction will proceed in stages: we first reconstruct the mathematical anatomy of Haller's derivation (Section 2), then define and analyze the Haller-Obidi Lagrangian and Action (Section 3), construct the covariant generalization (Section 4), prove the reduction from the Obidi Action (Section 5), explore the information-geometric and path-integral bridges (Sections 6–8), and delineate the limits of the correspondence (Section 9).

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2. Mathematical Anatomy of Haller's Derivation

This section provides a concise but complete mathematical reconstruction of the key derivations in Haller [1], focused exclusively on the mathematical structures that connect to the Theory of Entropicity. The historical and conceptual context — the entropy-as-generator tradition, the comparison with Bekenstein, Jacobson, Verlinde, and Frieden — was the subject of Letter IA and will not be repeated here. What follows is pure mathematical anatomy.

2.1 The Bernoulli-Gaussian Diffusion Model

Haller begins with a one-dimensional Bernoulli random walk as the fundamental model of quantum diffusion. A particle undergoes N independent steps, each of spatial length $\delta x = c\,\delta t$, where δt is the quantum time step (to be determined). At each step, the particle moves in the positive direction with probability β and in the negative direction with probability (1 – β). The average displacement after N steps is:

$$\bar{x}(N) = (2\beta - 1)\, \delta x \cdot N = (2\beta - 1)ct$$

where $t = N\,\delta t$. For a particle moving with average velocity v, the identification $\bar{x} = vt$ yields the kinematic relation:

$$(2\beta - 1) = v/c \quad \text{......(3)}$$

This equation maps the bias parameter β of the Bernoulli process directly onto the normalized velocity of the particle. A particle at rest corresponds to $\beta = 1/2$ (unbiased random walk); a particle moving at light speed corresponds to $\beta = 1$ (deterministic rightward motion). The entire velocity spectrum of special relativity is thus encoded in the single parameter $\beta \in [0, 1]$.

The variance of the Bernoulli walk after N steps, incorporating both the spatial position domain and the conjugate momentum domain through the de Broglie relation, takes the form:

$$(\Delta x_B(N))^2 = 2(1 - (v/c)^2)(\delta x)^2\, N \quad \text{......(4)}$$

The factor $(1 - v^2/c^2)$ is the Lorentz factor squared: the variance of the quantum diffusion decreases with velocity, vanishing at $v = c$. This is the stochastic signature of relativistic kinematics — faster particles diffuse less, and photons do not diffuse at all.

Haller then superimposes a Gaussian (minimum-uncertainty) wave packet on each step of the Bernoulli walk. Each step generates two eigenvalue states: $\Psi_+(x)$ centered at $+\delta x$ and $\Psi_-(x)$ centered at $-\delta x$, both with width Δx. The condition $\Delta x \ll \delta x$ ensures that the two Gaussian packets are non-overlapping, so the probability distributions add (not the amplitudes):

$$p_x(x) = \beta \cdot G(x; +\delta x, \Delta x) + (1 - \beta) \cdot G(x; -\delta x, \Delta x)$$

where $G(x; \mu, \sigma)$ is the Gaussian probability density with mean μ and standard deviation σ. The conjugate distribution $P_k(k/2\pi)$ in the frequency domain is obtained by Fourier transform. This Bernoulli-Gaussian structure is the microscopic model from which all subsequent information-theoretic quantities are computed. It combines discrete stochastic dynamics (the Bernoulli walk) with continuous quantum uncertainty (the Gaussian packet) — a structure that, as we shall demonstrate, maps onto the discrete-continuous duality of the entropic field in ToE.

2.2 The Hirshman Entropy Sum

Hirshman (1957) [9] proposed an entropic uncertainty relation based on the sum of differential entropies in conjugate domains. For a quantum state with position probability density $p(x)$ and momentum probability density $P(k)$, the Hirshman entropy sum is:

$$H_{\text{Hirshman}} = h(p) + h(P)$$

where $h(\cdot)$ denotes the differential (continuous) Shannon entropy. For the Bernoulli-Gaussian system constructed in Section 2.1, Haller evaluates this sum explicitly. The spatial differential entropy of the two-component Gaussian mixture has a contribution from the binary selection (which Gaussian center the particle occupies) and a contribution from the continuous spread of each Gaussian. The momentum-domain entropy contributes a corresponding term through the Fourier uncertainty relation $\Delta x \cdot \Delta k \geq 1/2$.

After careful evaluation — detailed in Haller [1, Section 3] and [21] — the total conditional entropy per Bernoulli step takes the compact form:

$$H_c = H_2(\beta) + \log(e/2) \qquad (5)$$

where $H_2(\beta) = -\beta \log \beta - (1 - \beta) \log(1 - \beta)$ is the binary Shannon entropy in natural units (nats). The term $\log(e/2)$ is the Hirshman contribution from the minimum-uncertainty Gaussian packet and is independent of the motion parameter β.

The critical quantization result follows immediately: when $\beta = 1/2$ (particle at rest), the binary entropy $H_2(1/2) = \log 2$, and the total conditional entropy per step is:

$$H_c(\beta = 1/2) = \log 2 + \log(e/2) = \log 2 + 1 - \log 2 = 1 \text{ nat}$$

This is the **entropic floor** of a quantum diffusion step: a particle at rest in a Bernoulli-Gaussian diffusion model produces exactly 1 natural unit of entropy per step. This quantization — $H_c = 1$ nat per step at $\beta = 1/2$ — is not imposed by hand; it emerges from the structure of the Hirshman entropy applied to the minimum-uncertainty Gaussian. Within the Theory of Entropicity, this quantization has a deep significance: it is the fundamental entropic quantum, the informational atom from which all larger entropic structures are composed.

2.3 Conditional Entropy Rate

Haller interprets the conditional entropy H_c as $H(\text{particle} \mid \text{vacuum})$ — the entropy of the particle state given the vacuum state. The particle's information content relative to the vacuum is measured by the rate at which this conditional entropy changes with time. To extract the rate, we Taylor-expand the binary entropy $H_2(\beta)$ around $\beta = 1/2$ for non-relativistic velocities $v/c \ll 1$.

From equation (3), $\beta = (1 + v/c)/2$. Expanding $H_2(\beta)$ to second order in v/c:

$$H_2(\beta) \approx \log 2 - (1/2)(v/c)^2/(\ln 2) \cdot \log e = \log 2 - (v/c)^2/2 + \mathcal{O}((v/c)^4)$$

The conditional entropy per step thus decreases with velocity — the particle becomes more predictable (less entropic) as its motion becomes more directed. The conditional entropy rate, obtained by dividing by the quantum time step $\delta t = \hbar/(2mc^2)$ (fixed by the de Broglie-Compton relation), becomes:

$$dH_C/dt = (2/\hbar)(mc^2 - K) \qquad (6)$$

where $K = ½mv^2$ is the non-relativistic kinetic energy. The quantum time step $\delta t = \hbar/(2mc^2)$ is the **Zitterbewegung period** — the timescale of intrinsic quantum jitter for a particle of mass m.

The information-theoretic interpretation of equation (6) is immediate and fundamental: **kinetic energy is the reduction in conditional entropy rate caused by directed motion.** A particle at rest produces conditional entropy at the maximum rate $(2/\hbar)mc^2$. As the particle accelerates, its entropy production decreases — the kinetic energy "uses up" some of the particle's informational capacity. At $v = c$, the conditional entropy rate would vanish (modulo relativistic corrections), and the particle would be informationally deterministic — a photon.

2.4 Mutual Information Rate

The conditional entropy captures the particle's intrinsic informational dynamics. The second component of the total entropy is the mutual information I_M between the particle and its environment (vacuum). Haller computes this using the Gaussian channel model from information theory [11].

The Gaussian channel has the form $Y = X + Z$, where X is the signal, Z is additive Gaussian noise independent of X, and Y is the received output. The channel capacity — the maximum mutual information — for a Gaussian input with power (variance) P and noise power N is:

$C = (1/2) \log(1 + P/N)$

Haller identifies the signal power with the Bernoulli variance: $S = (\Delta x_B)^2 = 2(1 - v^2/c^2)(\delta x)^2$ N, where the Bernoulli fluctuations represent the vacuum's "signal" to the particle. The noise power is identified with the thermal diffusion displacement: $N_D = v\hbar/F$, where F is the applied force and the thermal diffusion occurs over the relaxation timescale. After Taylor expansion in the non-relativistic limit and careful accounting for the positive and negative energy eigenvalue states (a Dirac-type two-component structure inherent in the Bernoulli model), the mutual information rate per unit time takes the form:

$dI_M/dt = (2/\hbar)V$ ....(7)

where V is the potential energy acting on the particle, with the baseline $V_0 = mc^2$ fixed by the independence condition: mutual information vanishes when the particle-vacuum interaction vanishes ($V = 0$, i.e., free particle). The factor of 2 from the Dirac-type positive/negative energy doubling is essential to obtaining the correct proportionality constant.

Equation (7) is the information-theoretic counterpart of equation (6). Where the conditional entropy rate encodes the kinetic energy, the mutual information rate encodes the potential energy. The physical interpretation is striking: **the potential energy experienced by a particle equals ($\hbar/2$ times) the rate at which the particle and its vacuum exchange mutual information.** Interaction is not a primitive mechanical concept — it is a rate of information exchange. A particle in a gravitational field is not "pulled" by a force; it is exchanging distinguishability with the vacuum at a rate proportional to the gravitational potential.

2.5 The Central Identity

The total self-information of the particle is the sum of conditional entropy and mutual information:

$H = H_C + I_M$

Taking the time derivative and using equations (6) and (7):

$dH/dt = (2/\hbar)(mc^2 - K) + (2/\hbar)V = (2/\hbar)(mc^2 - K + V) = (2/\hbar)(mc^2 - (K - V)) = (2/\hbar)(mc^2 - L)$

where $L = K - V$ is the classical Lagrangian. Integrating over time:

$H = (2/\hbar) \int (mc^2 - L)\, dt$ ....(8)

This is equation (1) restated with the full derivational context. It is the central identity of Haller's paper: the total Hirshman entropy of a quantum particle in a Bernoulli-Gaussian diffusion model equals the time integral of the mass-energy minus the classical Lagrangian, scaled by $2/\hbar$.

Rearranging equation (8) to isolate the classical action $S_{\text{action}} = \int L\, dt$:

$(\hbar/2)\, H = \int mc^2\, dt - S_{\text{action}}$ ....(9)

or equivalently:

$S_{\text{action}} = \int mc^2\, dt - (\hbar/2)\, H$ ....(10)

Equation (10) is the form most useful for the constructions of this Letter. It states: **the classical action equals the rest-energy time integral minus an entropy term, scaled by the quantum of action.** The action is not independent of entropy — it is the residue of the rest energy after the entropy has been subtracted.

A precise statement of what Haller proves is essential here. Haller does *not* claim "entropy equals action." He derives a more nuanced and more interesting identity: *the action equals the rest-energy baseline minus a scaled entropy.* Entropy contributes exactly where action lives — in the Lagrangian, in the variational principle, in the selection of classical trajectories. But the action is not the entropy; the action is the rest energy diminished by the entropy. This distinction will prove crucial when we construct the Haller-Obidi Lagrangian in the following section.

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3. The Haller-Obidi Action and Lagrangian — Definition and Construction

This section introduces the central new mathematical constructions of this Letter: the Haller-Obidi Lagrangian and the Haller-Obidi Action. These are obtained by rearranging Haller's entropy-action identity into forms that admit variational treatment and connect directly to the field-theoretic formulation of the Theory of Entropicity.

3.1 Defining the Haller-Obidi Lagrangian

We begin from Haller's entropy rate, obtained by differentiating equation (8) with respect to time:

$dH/dt = (2/\hbar)(mc^2 - L)$ ....(11)

This equation expresses the entropy rate as a linear function of the classical Lagrangian. We now perform the algebraic operation that transforms this information-theoretic identity into a variational object. Rearranging (11) to isolate the Lagrangian:

$L = mc^2 - (\hbar/2)(dH/dt)$ ....(12)

The right-hand side of (12) is a function of the particle's position, velocity, and time (through the entropy rate dH/dt, which depends on the particle's kinematic state). It therefore qualifies as a Lagrangian in the mechanical sense: a function on the tangent bundle of configuration space. We elevate this observation to a formal definition.

Definition 3.1 (Haller-Obidi Lagrangian).

The **Haller-Obidi Lagrangian** is the effective single-particle Lagrangian obtained by rearranging Haller's entropy-action identity:

$\mathcal{L}_{HO} \equiv mc^2 - (\hbar/2)\,\dot{H}$ ....(13)

where $\dot{H} \equiv dH/dt$ is the total entropy rate of the particle (sum of conditional entropy rate and mutual information rate).

This definition is not merely a notational rearrangement. It transforms Haller's information-theoretic result into a variational object with consequences that Haller himself did not pursue. The Lagrangian $\mathcal{L}_{HO}$ can be subjected to the Euler-Lagrange procedure:

$$d/dt\,(\partial\mathcal{L}_{HO}/\partial v) - \partial\mathcal{L}_{HO}/\partial x = 0$$

yielding equations of motion that are simultaneously mechanical (they determine trajectories) and informational (they extremize an entropy-related functional). The variational content of Haller's identity, latent in the original formulation, is made explicit by the Haller-Obidi Lagrangian. Every classical trajectory is an entropy-rate extremum; every entropy-rate extremum is a classical trajectory. The Haller-Obidi Lagrangian is the mathematical object that carries this dual character.

Physically, the structure of $\mathcal{L}_{HO}$ is transparent: the Lagrangian is the rest energy mc^2 diminished by the scaled entropy rate $(\hbar/2)\dot{H}$. A particle that produces entropy rapidly has a smaller effective Lagrangian; a particle that produces no entropy (a photon, in the Bernoulli limit $\beta \to 1$) has the maximum Lagrangian equal to mc^2. The entropy rate thus acts as an informational "cost" that reduces the effective action — the universe pays for entropy production by reducing the variational weight of the trajectory.

3.2 Defining the Haller-Obidi Action

The natural companion to the Haller-Obidi Lagrangian is its time integral — the action functional.

Definition 3.2 (Haller-Obidi Action).

The **Haller-Obidi Action** is the time integral of the Haller-Obidi Lagrangian over a trajectory from time t_1 to t_2:

$S_{\mathrm{HO}} \equiv \int_{t_1}^{t_2} \mathcal{L}_{\mathrm{HO}}\, dt = \int_{t_1}^{t_2} [mc^2 - (\hbar/2)\, \dot{H}]\, dt$ ....**(14)**

Evaluating the integral in (14) directly:

$S_{\mathrm{HO}} = \int_{t_1}^{t_2} mc^2\, dt - (\hbar/2) \int_{t_1}^{t_2} \dot{H}\, dt = \int_{t_1}^{t_2} mc^2\, dt - (\hbar/2)\, H$ ....**(15)**

where $H = H(t_2) - H(t_1)$ is the total entropy accumulated over the trajectory (assuming $H(t_1) = 0$ as initial condition, or equivalently, H represents the total accumulated entropy). Comparing (15) with equation (10):

$S_{\mathrm{action}} = \int mc^2\, dt - (\hbar/2)\, H$ ....(16)

we obtain the key result:

$S_{\mathrm{HO}} = S_{\mathrm{action}}$

$S_{\mathrm{HO}} = S_{\mathrm{action}}$ ....(17)

The Haller-Obidi Action *is* the classical action, rewritten in entropic variables.

This identity is exact, not approximate. The Haller-Obidi Action does not "approximate" the classical action or "resemble" it in some limit — it *is* the classical action, expressed in the language of entropy rates rather than kinetic and potential energies. The variational extremization of S_{HO} with respect to particle trajectories recovers the classical Euler-Lagrange equations of motion identically. But the variational principle is now transparently entropic: the classical path is the path that extremizes the integral of the rest energy minus the scaled entropy rate.

What equation (17) establishes is that the passage from Haller's information-theoretic identity to a variational action is not a separate construction — it is a tautology. The classical action was always the entropic action. Haller's derivation did not discover a new action; it revealed the informational content of the old one.

Why it is Called the Haller–Obidi Action

Even though the *Haller–Obidi Action* is still fundamentally **Haller's Action**, the name **Haller–Obidi Action** has been given because:

1. It denotes a *pair* of actions, not a single one.

"**Haller–Obidi Action**" has been used to mean the following two trajectories:

- **Haller's particle-level entropy–action identity**, and
- **the Obidi single-particle reduction of the Obidi Action**,

which coincide in the worldline limit.

So, the hyphen does **not** mean "a new action invented jointly by Haller and Obidi." It means (stands for):

Haller's Action as recovered from Obidi's Action.

Thus, the Haller–Obidi Action is simply the Haller Action identified as the single-particle sector of the full entropic field Obidi Action of the Theory of Entropicity (ToE). Consequently, the Haller–Obidi Lagrangian (as shown in subsequent sections of this Letter) is the Haller Lagrangian realized as the worldline-localized component of the Lagrangian density of the Obidi entropic field action.

3.3 Decomposition of the Haller-Obidi Lagrangian

The Haller-Obidi Lagrangian admits a natural decomposition that reflects the information-theoretic structure of Haller's derivation. Using the decomposition $H = H_C + I_M$ and the entropy rates from equations (6) and (7):

$\dot{H} = \dot{H}_C + \dot{I}_M = (2/\hbar)(mc^2 - K) + (2/\hbar)V$ ....(18)

Substituting (18) into the definition (13) of the Haller-Obidi Lagrangian:

$\mathcal{L}_{HO} = mc^2 - (\hbar/2)\,[(2/\hbar)(mc^2 - K) + (2/\hbar)V]$

$= mc^2 - (mc^2 - K) - V$

$= K - V$

$\mathcal{L}_{HO} = K - V = L$ ....(19)

This verifies internal consistency: the Haller-Obidi Lagrangian reduces identically to the classical Lagrangian $L = K - V$. The algebraic identity is exact and requires no approximation beyond those already present in Haller's derivation (non-relativistic Taylor expansion). But while equation (19) confirms consistency, the decomposition that produced it reveals the **informational anatomy of the classical Lagrangian**:

$\mathcal{L}_{HO} = \mathcal{L}_{free} + \mathcal{L}_{int}$ ....(20)

where:

$\mathcal{L}_{free} \equiv mc^2 - (\hbar/2)\,\dot{H}_C = K$ ....(21)

$\mathcal{L}_{int} \equiv -(\hbar/2)\,\dot{I}_M = -V$ ....(22)

The decomposition (20)–(22) is the informational anatomy of the classical Lagrangian, and it is the single most important structural result connecting Haller's framework to ToE's field-theoretic formulation:

(i) **The free Lagrangian arises from conditional entropy.** $\mathcal{L}_{\text{free}} = K = mc^2 - (\hbar/2)\dot{H}_C$. The kinetic energy is the difference between the rest energy and the conditional entropy rate — the particle's intrinsic informational dynamics, its entropy production relative to the vacuum. A free particle's Lagrangian is entirely conditional-entropic.

(ii) **The interaction Lagrangian arises from mutual information.** $\mathcal{L}_{\text{int}} = -V = -(\hbar/2)\dot{I}_M$. The potential energy (with sign) is the mutual information rate between particle and vacuum. Interaction — the coupling of a particle to an external field or potential — is the rate of information exchange between the particle and its environment.

This decomposition maps directly onto the standard field-theoretic decomposition of a Lagrangian density into free and interaction terms: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$. In quantum field theory, $\mathcal{L}_0$ describes the free propagation of quanta, and $\mathcal{L}_{\text{int}}$ describes their coupling. Haller's information-theoretic decomposition provides the entropic substrate of this algebraic split: free propagation is conditional entropy; interaction is mutual information. The classical Lagrangian is the informational residue of the entropic field evaluated along a worldline.

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4. Covariant Generalization — The Entropic Worldline Lagrangian

The Haller-Obidi Lagrangian (13) is a non-relativistic, non-covariant object: it uses the ordinary time derivative dH/dt, valid only in a preferred frame or in the non-relativistic limit $v/c \ll 1$. The Theory of Entropicity, by contrast, operates on a full pseudo-Riemannian manifold with a generally covariant entropic field $S(x)$. This section constructs the covariant generalization of the Haller-Obidi Lagrangian by embedding the entropy rate into the four-dimensional entropic field of the Theory of Entropicity (ToE).

4.1 From Time Derivative to Covariant Derivative

Haller's derivation uses dH/dt — the rate of change of the particle's total entropy with respect to coordinate time. In a generally covariant framework, coordinate time is not a scalar quantity; the natural time parameter along a particle worldline is the proper time τ, and the natural derivative operator along the worldline is the directional derivative along the four-velocity $u^\mu = dx^\mu/d\tau$.

The Theory of Entropicity posits an entropic density field $S(x)$ defined at every point of the spacetime manifold. This field is the fundamental dynamical variable of the theory — the primitive from which geometry, matter, and interactions emerge. Along a particle worldline $x^\mu(\tau)$, the rate of change of the entropic field with respect to proper time is the directional derivative:

$$dS/d\tau = u^\mu \, \partial_\mu S \quad \text{.......(23)}$$

This is a scalar quantity — invariant under general coordinate transformations — and reduces to dH/dt in the non-relativistic limit where $u^\mu \to (c, \mathbf{v})$ and $d\tau \to dt$. The replacement $dH/dt \to u^\mu \, \partial_\mu S$ is therefore the unique covariant lift of Haller's entropy rate. We use this to define the covariant Haller-Obidi Lagrangian.

Definition 4.1 (Covariant Haller-Obidi Lagrangian).

The **covariant generalization** of the Haller-Obidi Lagrangian along a timelike worldline $x^\mu(\tau)$ in the entropic field $S(x)$ is:

$$\mathcal{L}_{\text{ent}} = mc^2 - (\hbar/2)(u^\mu \, \partial_\mu S) \quad \text{.......(24)}$$

where $u^\mu = dx^\mu/d\tau$ is the four-velocity and $S(x)$ is the entropic density field of the Theory of Entropicity.

The covariant Lagrangian (24) has several important structural properties:

(i) **Reduction to $\mathcal{L}_{HO}$.** In the non-relativistic limit where $u^\mu \to (c, \mathbf{v})$, the spatial gradient terms are sub-dominant, and $u^\mu \partial_\mu S \to \partial_0 S + v^i \partial_i S \to dH/dt$. The covariant Lagrangian reduces to the Haller-Obidi Lagrangian (13) exactly.

(ii) **Coupling structure.** $\mathcal{L}_{ent}$ couples the particle's kinematic state (via u^μ) to the ambient entropic field (via $\partial_\mu S$). This coupling is of the minimal-coupling type: the entropic field gradient acts as an effective one-form that contracts with the four-velocity. The structure is analogous to the electromagnetic coupling $L_{em} = -(e/c)u^\mu A_\mu$ in which the electromagnetic potential A_μ contracts with the four-velocity. In the entropic case, the "potential" is $(\hbar/2)$ $\partial_\mu S$.

(iii) **Scalar character.** Since $S(x)$ is a scalar field and $u^\mu \partial_\mu S$ is a scalar (the contraction of a vector and a co-vector), $\mathcal{L}_{ent}$ is invariant under general coordinate transformations. The covariant Haller-Obidi Lagrangian is a legitimate Lagrangian on any pseudo-Riemannian manifold $(M, g_{\mu\nu})$.

4.2 The Entropic Current and Continuity

The covariant Lagrangian (24) naturally introduces the concept of an entropic current. For a distribution of particles (or, more generally, for a continuous entropic medium), the current of entropic density is:

$J^\mu_S = \rho_S u^\mu$ ....(25)

where ρ_S is the entropic density (entropy per unit three-volume) and u^μ is the four-velocity field of the entropic fluid. If the entropic field satisfies a conservation law — the **Obidi Principle of Conserved Entropic Flux (OPCEF)** — then the entropic current is divergence-free:

$\nabla_\mu J^\mu_S = 0$ ....(26)

This equation expresses the conservation of total entropic flux: while entropy may redistribute itself within the manifold (flowing from one region to another, converting between conditional and mutual forms), the total integrated entropic flux through any closed hypersurface vanishes. This is a stronger statement than the second law of thermodynamics, which only constrains the sign of entropy production. Equation (26) constrains the *total flux*, not just the local production.

Simultaneously, the second law — or more precisely, ToE's principle that entropic evolution is irreversible along any timelike worldline — requires:

$$u^\mu \partial_\mu S \geq 0 \qquad (27)$$

Equations (26) and (27) are compatible: the flux is globally conserved while the local entropy density along any worldline is non-decreasing. The physical picture is that of an incompressible entropic fluid — the total amount of "entropic substance" is fixed, but it flows irreversibly in the direction of increasing proper time. This covariant structure — conserved flux with irreversible local evolution — is the defining feature of ToE's entropic dynamics. It is absent from Haller's non-covariant framework, where only the entropy rate dH/dt is defined, with no notion of a conserved current or a divergence-free flux. The covariant Haller-Obidi Lagrangian (24) thus serves as the entry point through which these field-theoretic structures enter the particle-level formalism.

4.3 The Covariant Entropic Action

The covariant Haller-Obidi Action is obtained by integrating $\mathcal{L}_{\text{ent}}$ over the proper time along the worldline:

$$S_{\text{ent}} = \int \mathcal{L}_{\text{ent}}\, d\tau = \int [mc^2 - (\hbar/2)(u^\mu \partial_\mu S)]\, d\tau \qquad (28)$$

This action is a functional of two objects: the worldline $x^\mu(\tau)$ (through u^μ) and the entropic field $S(x)$ (through $\partial_\mu S$). Holding $S(x)$ fixed as a background field and varying with respect to the worldline, we obtain the equations of motion for a particle propagating in the entropic field. The variation $\delta S_{ent}/\delta x^{\mu} = 0$ yields, schematically:

$$m(du^{\mu}/d\tau) = (\hbar/2)\,[\partial^{\mu}(u^{\nu}\,\partial_{\nu} S) - u^{\mu}\,u^{\nu}\,\partial_{\nu}(\partial_{\alpha} S)\,g^{\alpha\mu}/c^2\,] \quad \text{...(29)}$$

The precise form of (29) depends on the functional form of $S(x)$ and requires specification of boundary conditions and the constraint $u^{\mu}\,u_{\mu} = -c^2$. The schematic structure, however, is transparent: **the gradient of the entropic field acts as an effective force on the particle.** The "entropic force" $(\hbar/2)\,\partial^{\mu}(u^{\nu}\,\partial_{\nu}\,S)$ deflects geodesics in proportion to the entropy gradient — regions of high entropy gradient curve worldlines, just as regions of high spacetime curvature curve geodesics in general relativity. This is the covariant entropic mechanics that emerges from marrying Haller's information-theoretic particle identity with ToE's field ontology.

The structure of the entropic force term in (29) deserves comparison with the geodesic deviation in general relativity. In Einstein's theory, the deviation of a worldline from a geodesic is governed by the Riemann curvature tensor: $m(du^{\mu}/d\tau) = F^{\mu}_{ext}$. In ToE, the entropic field gradient provides an additional source of worldline deviation, even in flat spacetime. If the entropic field $S(x)$ and the metric $g_{\mu\nu}$ are dynamically coupled (as they are in the full Obidi Action), then the distinction between "gravitational" and "entropic" forces dissolves — both are manifestations of the entropic field's geometry. This unification is the subject of the full ToE formulation and lies beyond the scope of the single-particle analysis; but the covariant Haller-Obidi Action (28) is the precise point at which the particle-level and field-level descriptions make contact.

5. Reduction of the Obidi Action to the Haller-Obidi Action

We now establish the central **structural theorem** of this Letter: ***the Obidi Action of the Theory of Entropicity (ToE) reduces to the Haller-Obidi Action upon localization of the entropic field to a single timelike worldline.*** This reduction establishes the Obidi-Haller Correspondence as a rigorous mathematical limit.

Theorem 5.1 (Haller-Obidi Structural Theorem). (Structural Reduction).

The Obidi Action of the Theory of Entropicity (ToE) reduces to the Haller-Obidi Action upon localization of the entropic field to a single timelike worldline.

This expresses the formal reduction — the mathematical statement that the Obidi Action reduces to the Haller–Obidi Action under worldline localization. It's a **structural theorem**, not a correspondence. It provides the explicit reduction chain:

S_Obidi→S_ent→S_action,

and the conversion H=(2/ℏ)S_HO.

That is:

$$S_{\mathbf{Haller}} = S_{\mathbf{Obidi}} \mid_{\mathbf{worldline\ localization}}$$

Note:

Theorem 5.1 formalizes the reduction underlying Proposition 5.1, establishing the structural basis of the Obidi–Haller Correspondence.

5.1 The Obidi Action

The Obidi Action is the fundamental action functional of the Theory of Entropicity (ToE), defined on the entropic manifold $(M, g_{\mu\nu})$ with the entropic density field $S(x)$ as the fundamental dynamical variable:

$$S_{\text{Obidi}} = \int_M F(S, \nabla S, g_{\mu\nu}) \sqrt{(-g)}\, d^4x \quad (30)$$

Here F is the entropic Lagrangian density — a scalar function of the entropic field, its first (and possibly second) covariant derivatives, and the metric. The variation $\delta S_{\text{Obidi}}/\delta S = 0$ yields the **Obidi Field Equations** (Master Entropic Equation):

$$\partial F/\partial S - \nabla_\mu(\partial F/\partial(\partial_\mu S)) + \nabla_\mu \nabla_\nu(\partial F/\partial(\partial_\mu \partial_\nu S)) = 0 \quad (31)$$

The Obidi Field Equations (OFE) are a second-order (or, if the Lagrangian density depends on $\partial_\mu \partial_\nu S$, fourth-order) partial differential equation for the entropic field $S(x)$ on the manifold M. These equations govern the full dynamics of the entropic field — including the emergence of spacetime geometry (via the coupling to $g_{\mu\nu}$), the propagation of entropic waves, and the formation of localized entropic configurations (particles).

The Obidi Action (30) is a field-theoretic object — it integrates over the entire four-dimensional manifold. Haller's entropy-action identity (1), by contrast, is a particle-level object — it integrates along a single one-dimensional worldline. The question is: how does the former reduce to the latter?

5.2 Worldline Localization

The reduction proceeds through a **worldline localization** of the entropic field. Consider an entropic field configuration that is sharply concentrated along a single timelike worldline $x^\mu(\tau)$:

$$S(x) \to S_0(\tau) \cdot \delta^3(x - x(\tau)) \,/\, \sqrt{(-g)} \quad (32)$$

where $S_o(\tau)$ is the entropy profile along the worldline and δ^3 is the three-dimensional covariant delta function transverse to the worldline. This ansatz describes a "particle" — a localized lump of entropic density propagating along the worldline $x^\mu(\tau)$. The entropic field away from the worldline vanishes; all the entropy is concentrated on the trajectory itself.

Substituting the localized ansatz (32) into the Obidi Action (30) and performing the integration over the three spatial dimensions transverse to the worldline:

$$S_{\text{Obidi}} \rightarrow \int F_{\text{eff}}(S_o, dS_o/d\tau)\, d\tau \quad (33)$$

The four-dimensional integral collapses to a one-dimensional integral along the worldline. The effective Lagrangian F_{eff} is obtained by evaluating the entropic Lagrangian density F on the localized configuration and integrating out the transverse delta function.

To make the reduction explicit, we need to specify the form of the entropic Lagrangian density F. The minimal ansatz consistent with the symmetries of ToE — scalar under diffeomorphisms, at most first-order in derivatives of S, linear coupling between the entropic gradient and the four-velocity field — takes the form:

$$F = \varepsilon_o(S) - (\hbar/2)\, g^{\mu\nu}\, (\partial_\mu S)(u_\nu) + F_{\text{int}} \quad (34)$$

where $\varepsilon_o(S)$ is the entropic self-energy density (a function of the field value alone), the second term is the kinetic-entropic coupling, and F_{int} contains higher-order interaction terms. Upon worldline localization (substituting (32) into (34) and integrating transversely), the effective one-dimensional action becomes:

$$S_{\text{Obidi}}^{(1)} = \int [\varepsilon_o(S_o) - (\hbar/2)(u^\mu\, \partial_\mu\, S_o)]\, d\tau \quad (35)$$

The identification of $\varepsilon_o(S_o)$ with mc^2 for a localized configuration of mass m is the statement that the rest energy of a particle is its entropic self-energy — the energy stored in the localized entropic configuration itself. This identification is not ad hoc; it is the particle-level expression of the general principle that mass-energy is frozen entropy (a central tenet of ToE). With this identification:

$$S_{\text{Obidi}}^{(1)} = \int [mc^2 - (\hbar/2)\,\dot{S}_0]\, d\tau = S_{\text{ent}} \quad (36)$$

This is precisely the covariant Haller-Obidi Action (28). In the non-relativistic limit:

$$S_{\text{Obidi}}^{(1)} \to S_{\text{HO}} = \int [mc^2 - (\hbar/2)\,\dot{H}]\, dt = S_{\text{action}} \quad (37)$$

which is Haller's original entropy-action identity. The reduction is complete.

5.3 The Obidi-Haller Correspondence (Formal Statement)

Proposition 5.1 (Obidi-Haller Correspondence). (Conceptual Correspondence)

Let $S_{\text{Obidi}} = \int_M F(S, \nabla S, g_{\mu\nu}) \sqrt{(-g)}\, d^4x$ be the Obidi Action defined on the entropic manifold M. Under worldline localization of the entropic field $S(x)$ to a single timelike trajectory $x^\mu(\tau)$, the Obidi Action reduces to the covariant Haller-Obidi Action:

$$S_{\text{Obidi}} \to S_{\text{ent}} = \int \mathcal{L}_{\text{ent}}\, d\tau \quad (38)$$

In the non-relativistic limit ($v \ll c$), the covariant Haller-Obidi Action further reduces to the classical action:

$$S_{\text{ent}} \to S_{\text{action}} = \int L\, dt \quad (39)$$

with the conversion:

$$H = (2/\hbar)\, S_{\text{HO}} \quad (40)$$

reproducing Haller's entropy-action identity exactly.

Note:

Theorem 5.1 formalizes the reduction underlying Proposition 5.1, establishing the structural basis of the Obidi–Haller Correspondence.

Proposition 5.1 establishes the following hierarchy of actions in ToE:

Level	Action	Domain	Dynamical Variable
Field level	$S_{\text{Obidi}} = \int F \sqrt{(-g)}\, d^4x$	Full manifold M	Entropic field $S(x)$
Worldline level	$S_{\text{ent}} = \int \mathcal{L}_{\text{ent}}\, d\tau$	Single worldline	Worldline $x^\mu(\tau)$
Non-relativistic	$S_{\text{HO}} = S_{\text{action}} = \int L\, dt$	Single trajectory	Path $x(t)$

Item	Concept	Naming	Logic
1	**Action pair**	**Haller–Obidi Action**	Haller's action recovered from Obidi's reduction
2	**Structural theorem**	**Haller–Obidi Structural Theorem**	Reduction ends in Haller's action
3	**Correspondence**	**Obidi–Haller Correspondence**	Interpretation begins with Obidi's ontology

Each level emerges from the one above by restriction. The Obidi Action contains the Haller-Obidi Action as a sector; the Haller-Obidi Action contains the classical action as a limit. This nesting structure is the mathematical content of the claim that Haller's entropy-action identity is the "single-particle projection" of the Obidi Action: a projection of the full field-theoretic dynamics onto a one-dimensional worldline, followed by a non-relativistic truncation.

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6. Mutual Information as Entropic Potential — Toward Information-Geometric Coupling

Sections 3–5 established the structural connection between Haller's entropy-action identity and the Obidi Action through the Haller-Obidi Lagrangian and its covariant generalization. This section explores a different bridge: the connection between Haller's mutual information and the information-geometric structures of ToE. The key equation is (7): $dI_M/dt = (2/\hbar)V$.

6.1 The Information-Interaction Correspondence

Rearranging equation (7) to express the potential energy in terms of the mutual information rate:

$V = (\hbar/2)\, \dot{I}_M$ ....(41)

Equation (41) is a statement of the **information-interaction correspondence**: the potential energy experienced by a particle is not a mechanically primitive quantity but an information-exchange rate. The potential energy between two systems equals $\hbar/2$ times the rate at which they exchange mutual information.

The implications of this correspondence for the Theory of Entropicity are far-reaching. In standard physics, the potential energy $V(x)$ is typically specified as an input to the Lagrangian — it encodes the external field or the inter-particle interaction, and its functional form (Coulomb, gravitational, harmonic, etc.) is determined empirically or by symmetry arguments. Equation (41) provides an information-theoretic origin for this input: the potential is not a free function but is constrained to equal the mutual information rate between the particle and its environment. Any physically realizable potential must correspond to an achievable mutual information rate in the underlying information channel between particle and vacuum.

This constraint is non-trivial. The mutual information rate is bounded by the channel capacity, which depends on the signal-to-noise ratio of the particle-vacuum channel. For a Gaussian channel (Haller's model), the capacity is $(1/2)\log(1 + P/N)$ nats per use. The potential energy is therefore bounded by:

$V \leq (\hbar/2) \cdot (1/\delta t) \cdot (1/2) \log(1 + P/N) = (mc^2/2) \log(1 + P/N)$

This is an information-theoretic bound on the potential energy — a "channel capacity bound" on interactions. Within ToE, this bound acquires physical significance: it constrains the maximum interaction strength at a given energy scale, potentially connecting to the ultraviolet structure of quantum field theory.

6.2 Entropic Coupling Constants

Equation (41) suggests a natural parameterization of the mutual information rate that leads to the concept of entropic coupling constants. If the mutual information rate can be decomposed as:

$\dot{I}_M = g_{ent} \cdot f(x, v)$ ....(42)

where g_{ent} is a dimensionless entropic coupling constant and $f(x, v)$ encodes the geometric and kinematic dependence, then the potential energy takes the form:

$V = (\hbar/2)\, g_{ent}\, f(x, v)$ ....(43)

Different functional forms of f correspond to different physical interactions:

Interaction	**Potential $V(r)$**	**Mutual Information Mode $f(x)$**	**Coupling g_{ent}**
Gravitational	$-GMm/r$	$f \sim 1/r$	$g_{grav} \sim GM/(\hbar c)$
Coulomb	$e^2/(4\pi\varepsilon_0 r)$	$f \sim 1/r$	$g_{em} \sim \alpha_{em}$
Harmonic	$\frac{1}{2}kx^2$	$f \sim x^2$	$g_{harm} \sim k/(\hbar\omega)$

The entropic coupling constant g_{ent} measures the "informational strength" of an interaction — the efficiency with which the particle and vacuum exchange distinguishability. The fine structure constant $\alpha_{em} \approx 1/137$, in this interpretation, is not a mysterious dimensionless parameter but a measure of the mutual information rate per unit geometric factor for electromagnetic interactions. This identification provides a concrete route for computing coupling constants from information-theoretic first principles within the full ToE framework.

6.3 From Mutual Information to Effective Metric

Mutual information is not only a measure of statistical dependence; it is also a natural generator of geometric structure. In information geometry (Amari and Nagaoka [16]), the mutual information between a parametric family of distributions $p(\cdot;\,\theta)$ and a reference distribution induces a Riemannian metric on the parameter space — the Fisher-Rao metric:

$$g_{\mu\nu}^{(FR)}(\theta) = \mathbb{E}[\partial_\mu \log p \cdot \partial_\nu \log p]$$

Haller's mutual information rate $\dot{I}_M = (2/\hbar)V$ involves a parametric family of vacuum states parameterized by the particle's phase-space coordinates $\theta^\mu = (x,\ v,\ \ldots)$. The second derivative of the mutual information with respect to these parameters defines an effective metric on the space of particle-vacuum configurations:

$$g_{\mu\nu}^{(ent)} = \partial^2 I_M \,/\, \partial\theta^\mu\, \partial\theta^\nu \quad (44)$$

This is the Fisher-Rao metric on the space of vacuum configurations, evaluated through the lens of particle-vacuum mutual information. Within the Theory of Entropicity, this Fisher-Rao metric is precisely the emergent physical metric — the $g_{\mu\nu}$ that appears in the Obidi Action (30) — in the $\alpha = 0$ sector of the entropic α-connection. Haller's mutual information thus provides a constructive route from information-theoretic quantities to the geometric structures of ToE: the metric of spacetime is the curvature of mutual information.

The identification (44) also provides a concrete mechanism for the "emergence of geometry from entropy" that ToE postulates. The metric is not a fundamental object — it is derived from the second-order structure of the mutual information between the entropic field and its localized excitations. Flat spacetime corresponds to constant mutual information (no information-geometric curvature); curved spacetime corresponds to position-dependent mutual information (information-geometric curvature proportional to the gradient of the mutual information rate). The Haller framework, limited to a single particle in flat space, probes only the trivial (flat) limit of this correspondence — but the mathematical structure is already in place, waiting for the full ToE treatment.

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7. The Gaussian Channel and the Entropic α-Connection

Haller's mutual information calculation relies critically on the Gaussian channel model — the information-theoretic channel that maximizes mutual information for a given signal-to-noise ratio. This section identifies the Gaussian channel as a specific sector of the more general information-geometric framework of ToE, and explores the implications of this identification.

7.1 Haller's Gaussian Channel as α = 0 Sector

In Amari's (and Čencov's) information geometry [16], a statistical manifold — a space of probability distributions — admits a one-parameter family of affine connections called the α-connections. The parameter $\alpha \in \mathbb{R}$ indexes the "type" of geometry:

α Value	Connection Type	Distribution Family	Physical Sector
$\alpha = 0$	Levi-Civita (metric-compatible, torsion-free)	Gaussian (normal)	Standard quantum mechanics
$\alpha = 1$	Exponential connection	Exponential family (Boltzmann-Gibbs)	Classical thermodynamics
$\alpha = -1$	Mixture connection	Mixture family	Bayesian inference
General α	α-connection	Rényi/Tsallis distributions	Non-extensive systems

The Gaussian distribution is the unique distribution that simultaneously belongs to the exponential family ($\alpha = 1$) and the mixture family ($\alpha = -1$), and its Fisher-Rao geometry coincides with the Levi-Civita connection ($\alpha = 0$). The $\alpha = 0$ sector is the sector of minimum information-geometric curvature — the "flat" limit of the statistical manifold.

Haller's entire derivation operates within this Gaussian/Levi-Civita sector. His wave packets are Gaussian; his channel model is Gaussian; his entropy calculations use the Gaussian form of the differential entropy. The correspondence is therefore:

Haller's Gaussian channel $\leftrightarrow$ $\alpha = 0$ sector of ToE's entropic α-connection ....(45)

This correspondence has immediate and far-reaching implications. It means that Haller's entropy-action identity is valid within the minimum-curvature sector of the entropic manifold — the sector where the Fisher-Rao metric coincides with the Levi-Civita connection, where the information-geometric curvature is entirely Riemannian, and where the statistical mechanics is Boltzmann-Gibbs. This is the sector that describes standard quantum mechanics, ordinary thermodynamics, and weak-field gravity — precisely the regime where Haller's non-relativistic Taylor expansion is valid.

Extensions beyond the Gaussian channel would probe other α-sectors of ToE's information geometry. Non-Gaussian channels (Lévy flights, Cauchy distributions, heavy-tailed processes) correspond to $\alpha \neq 0$, where the information-geometric connection acquires torsion ($\alpha \neq 0$) and the entropy generalizes from Shannon to Rényi or Tsallis forms. This suggests that anomalous diffusion processes, non-extensive thermodynamics (Tsallis statistics), and strongly correlated quantum systems (non-Gaussian quantum states) correspond to different curvature sectors of the same entropic manifold. The Haller-Obidi construction, living in the $\alpha = 0$ sector, is the zeroth-order approximation; the full ToE framework accesses the entire α-spectrum.

7.2 The Hirshman Entropy as Entropic Floor

The Hirshman entropy sum — the sum of differential entropies in position and momentum space — achieves its minimum value for the Gaussian (minimum-uncertainty) wave packet. From Section 2.2, the Hirshman entropy per step at $\beta = 1/2$ is exactly 1 nat. This quantization is not a consequence of a discretization choice or a regularization scheme; it emerges from the structure of the Hirshman entropy applied to the minimum-uncertainty state.

Within the Theory of Entropicity (ToE), this quantization acquires the status of a fundamental boundary condition: the **entropic floor**. The entropic floor is the minimum entropy production per quantum step — the smallest unit of informational change in the entropic manifold. No physical process can produce less than 1 nat of Hirshman entropy per quantum time step $\delta t = \hbar/(2mc^2)$.

The entropic floor provides a boundary condition for the Obidi Field Equations (31): no solution of the OFE can produce a single-particle entropy below 1 nat per quantum step. This constraint is equivalent to the Heisenberg uncertainty principle $\Delta x \cdot \Delta p \geq \hbar/2$, but expressed in purely information-theoretic language. The uncertainty principle is not a restriction on simultaneous measurements; it is a lower bound on the entropy production rate of a quantum system. The Hirshman floor is the information-theoretic origin of quantum uncertainty, rendered as a curvature bound on the entropic manifold.

This identification sharpens the role of $\hbar$ in the Haller-Obidi construction. The reduced Planck constant appears in the Haller-Obidi Lagrangian (13) as the conversion factor between entropy rate and energy: $\mathcal{L}_{\mathrm{HO}} = mc^2 - (\hbar/2)\dot{H}$. The factor $\hbar/2$ is not arbitrary — it is fixed by the Hirshman floor: 1 nat of entropy per step of duration $\hbar/(2mc^2)$ corresponds to an energy of mc^2. The Planck constant is thus the ratio of the rest energy to the entropic floor rate — the quantum of entropic capacity. In ToE, $\hbar$ is not a fundamental constant of nature but a derived quantity: $\hbar = 2mc^2\ \delta t = 2 \times$ (entropic self-energy) $\times$ (entropic time step). The Haller-Obidi construction, by making this relationship explicit, provides the clearest available example of the derivability of $\hbar$ from entropic principles.

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8. Entropy-Weighted Path Selection and the Vuli-Ndlela Bridge

Haller's result that least action equals maximum entropy implies that the path selection principle of classical mechanics is, at its core, an entropy optimization. This section explores the implications of this identification for the path integral formulation of quantum mechanics and for the Vuli-Ndlela Integral (VNI) of the Theory of Entropicity (ToE).

8.1 From Least Action to Entropy-Weighted Histories

In the Feynman path integral formulation of Quantum Field Theory (QFT), the quantum-mechanical propagator between two spacetime points is given by a sum over all possible paths connecting them, with each path weighted by the phase factor $\exp(iS[x]/\hbar)$:

$K = \int \mathcal{D}[x] \exp(iS[x]/\hbar)$ ....(46)

The action $S[x] = \int L\, dt$ determines the phase of each path's contribution. Paths near the classical trajectory (where $\delta S = 0$) contribute constructively; paths far from the classical trajectory contribute with rapidly oscillating phases that cancel in the stationary-phase approximation.

Haller's identification $S_{\text{action}} = \int mc^2\, dt - (\hbar/2)H$ from equation (10) means that the action weighting can be rewritten in terms of entropy. Substituting into (46):

$K = \int \mathcal{D}[x] \exp\{i[\int mc^2\, dt - (\hbar/2)H[x]]/\hbar\} = e^{i\int mc^2 dt/\hbar} \int \mathcal{D}[x] \exp(-iH[x]/2)$ ....(47)

Up to the constant phase factor $e^{i\int mc^2 dt/\hbar}$ (which cancels in transition amplitudes), the path integral is a sum over paths weighted by $\exp(-iH[x]/2)$. The action weighting *is* an entropy weighting. The Feynman path integral already contains, implicitly, a preference for paths that extremize entropy — but this preference is encoded in the oscillatory phase, not in a real exponential suppression. The classical limit ($\hbar \to 0$) selects the path of extremal action = extremal entropy, recovering the classical trajectory.

8.2 The Vuli-Ndlela Integral (VNI)

The Theory of Entropicity (ToE) generalizes the Feynman path integral by introducing an explicit entropy-selective weighting. The **Vuli-Ndlela Integral (VNI)** is defined (in its most elementary fashion) as:

$$Z_{\mathrm{VN}} = \int \mathcal{D}[x] \exp\{iS[x]/\hbar\} \cdot W_{\mathrm{ent}}[S(x)] \quad \text{...(48)}$$

where W_{ent} is an entropic weighting functional — a real-valued, non-negative functional of the entropic field evaluated along the path. The simplest parameterization of the entropic weighting takes the exponential form:

$$Z_{\mathrm{VN}} = \int \mathcal{D}[x] \exp\{iS[x]/\hbar + \lambda H[x]\} \quad \text{...(49)}$$

where λ is the **entropic selection parameter** — a dimensionless real parameter that controls the strength of the entropy weighting relative to the action weighting.

The Vuli-Ndlela Integral has three distinguished limits:

(i) **$\lambda = 0$:** The entropic weighting is trivial ($W_{\mathrm{ent}} = 1$), and the Vuli-Ndlela Integral reduces to the standard Feynman path integral (46). Standard quantum mechanics is the $\lambda = 0$ sector of the Vuli-Ndlela Integral — the sector in which entropy influences path selection only through the action (implicitly, via Haller's identity) and not through an additional explicit weighting.

(ii) **$\lambda > 0$:** Paths with higher entropy production are preferentially selected. The $\exp(\lambda H)$ factor amplifies the contribution of high-entropy paths relative to low-entropy paths, introducing an *intrinsic time asymmetry* into the path integral. The Feynman path integral is time-symmetric (it weights forward and backward paths equally); the Vuli-Ndlela Integral with $\lambda > 0$ is time-asymmetric (it prefers the forward direction, the direction of entropy increase). This is the Vuli-Ndlela mechanism for the arrow of time: the second law of thermodynamics is not imposed on quantum mechanics from outside but is built into the path integral as the $\lambda > 0$ condition.

(iii) **$\lambda \to \infty$:** The entropy weighting dominates completely, and the Vuli-Ndlela Integral selects only the path of maximum entropy production — the thermodynamic extremum. This is the classical thermodynamic limit, where the variational principle is purely entropic and the action plays no role.

Haller's result provides the essential bridge between the standard path integral and the Vuli-Ndlela generalization. By showing that $S[x]$ and $H[x]$ are linearly related for a single particle (equation (10)), Haller demonstrates that the standard Feynman path integral already contains an implicit entropy weighting. The Vuli-Ndlela Integral makes this implicit structure explicit and generalizes it: the parameter λ promotes the implicit entropic content of the action to an independent degree of freedom, allowing the entropy weighting to decouple from the action weighting at high energies, strong fields, or near the Planck scale.

8.3 The Entropic Selection Principle

The Vuli-Ndlela parameter λ encodes what the Theory of Entropicity (ToE) calls the **entropic selection principle (ESP)**: among all histories compatible with the boundary conditions, the universe selects those that jointly extremize both the action and the entropy production. The selection functional is:

$$\Gamma[x] = S[x]/\hbar + \lambda H[x]$$

and the selected history satisfies $\delta\Gamma/\delta x^\mu = 0$, which gives:

$$(1/\hbar)\ \delta S/\delta x^\mu + \lambda\ \delta H/\delta x^\mu = 0$$

In the Haller regime, where $S = \int mc^2\, dt - (\hbar/2)H$, the two variations are proportional and the entropic selection principle reduces to the standard variational principle with a renormalized effective action. Outside the Haller regime — at high energies, where the linear relation between action and entropy breaks down, or in the vicinity of entropic phase transitions, where the entropic field undergoes rapid restructuring — the two terms in $\delta\Gamma$ are no longer proportional, and the entropic selection parameter λ introduces genuine corrections to the path integral.

These corrections are, in principle, observable. The Vuli-Ndlela parameter λ modifies transition amplitudes, scattering cross-sections, and decay rates by a factor proportional to the entropy difference between initial and final states. Processes that produce large entropy differences (irreversible processes, thermalization, black hole formation) receive larger corrections than processes that produce small entropy differences (elastic scattering, adiabatic evolution). The experimental signature of the entropic selection principle is therefore a correlation between the irreversibility of a process and the deviation of its quantum-mechanical transition amplitude from the standard Feynman prediction — a deviation that vanishes in the $\lambda \to 0$ limit and grows with the entropic asymmetry of the process.

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9. Limits of the Haller-ToE Correspondence

The preceding sections have established extensive mathematical connections between Haller's 2015 entropy-action identity and the entropic field theory of ToE. Intellectual honesty requires an equally precise delineation of where the correspondence ends — what structures of ToE have no counterpart in Haller's framework, and what assumptions of Haller's framework do not survive the passage to the full field theory.

9.1 What Haller Does Not Provide

The following elements of the Theory of Entropicity are absent from Haller's 2015 paper, and cannot be derived from or reduced to his framework:

(i) No entropic field. Haller works with entropy rates and mutual information evaluated for a specific particle-vacuum system in a Bernoulli-Gaussian diffusion model. He does not define a universal entropy field $S(x)$ — a scalar function on the spacetime manifold whose dynamics generate geometry and matter. His framework is entropy-informed mechanics: it uses entropy to reinterpret the classical action, but it does not elevate entropy to the status of a fundamental dynamical field. The entropic field $S(x)$ of ToE, with its own equation of motion (31), conserved current (25)–(26), and coupling to the metric $g_{\mu\nu}$, is a structure without analog in Haller's paper.

(ii) No conserved entropic flux. The **Obidi Principle of Conserved Entropic Flux (OPCEF)** — the statement $\nabla_\mu J^\mu{}_S = 0$ — is a central axiom of the Theory of Entropicity. Haller does not derive, state, or use anything of this form. His entropy $H(t)$ is a monotonically increasing function of time (by the second law), but there is no four-current, no divergence condition, and no notion of conserved flux. The covariant conservation law (26) is a ToE construction imposed on the Haller-Obidi Action from above, not derived from within Haller's framework.

(iii) No intrinsic time asymmetry. Haller appeals to the second law of thermodynamics and entropy increase in motivating his derivation, and his entropy rates are positive by construction. But he does not formulate an intrinsic mechanism for time asymmetry at the level of the path integral or the variational principle. The Vuli-Ndlela Integral (49) with $\lambda > 0$, which encodes time asymmetry as a preferential selection of high-entropy paths, is a ToE construction. Haller's time arrow is the conventional thermodynamic arrow — it distinguishes past from future by entropy increase — but it is not built into the action principle itself.

(iv) No field equations. Haller derives an identity between entropy and action but does not construct dynamical equations governing the evolution of an entropic field. The Obidi Field Equations (OFE) (31), which determine how the entropic field [written elementarily as] $S(x)$ propagates, interacts, and couples to the metric, have no counterpart in Haller's framework. His result is kinematic (an identity between integrated quantities), not dynamic (an equation of motion for a field).

(v) Non-relativistic limitation. Haller's derivation relies on Taylor expansions in $v/c \ll 1$ at several critical steps — the expansion of the binary Shannon entropy $H_2(\beta)$, the non-relativistic kinetic energy $K = \frac{1}{2}mv^2$, and the thermal diffusion model. The resulting identity (1) is exact only in the non-relativistic limit. The covariant generalization (Section 4) extends the result to arbitrary velocities and curved spacetimes, but this extension is a ToE construction that goes beyond what Haller's paper establishes.

9.2 What Haller Does Provide

Despite the limitations catalogued above, Haller's 2015 paper provides several structures of direct and substantive value to the Theory of Entropicity (ToE):

(i) A concrete entropy-action equivalence. Haller derives, from independently motivated information-theoretic premises, a mathematical identity between entropy and the classical action. This gives ToE an independently derived, published, peer-reviewed, worked example of the entropy-action correspondence at the particle level — a result that ToE can absorb as a sector of its broader framework without circular reasoning.

(ii) A free-plus-interaction decomposition. The decomposition $H = H_C + I_M$, corresponding to $L = K - V$ via the Haller-Obidi Lagrangian (equations (20)–(22)), maps naturally onto the standard field-theoretic Lagrangian decomposition $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$. This provides a concrete information-theoretic interpretation of the free-interaction split that ToE can leverage in its field-level formulation.

(iii) Potential energy as mutual information. The identification $V = (\hbar/2)\dot{I}_{\mathrm{M}}$ (equation (41)) is a constructive bridge between information theory and interaction physics. It demonstrates that at least one class of physical potentials (those arising from Gaussian channel particle-vacuum interactions) admits a rigorous mutual-information interpretation — providing a prototype for the information-geometric potentials of ToE.

(iv) Quantization of entropy. The Hirshman floor — 1 nat per quantum step at $\beta = 1/2$ — is an independent derivation of entropy quantization that resonates with ToE's prediction that the entropic field has a minimum quantum of change. The convergence of two independent arguments (Haller's information-theoretic derivation and ToE's field-theoretic prediction) for the same quantization condition strengthens both.

(v) An explicit link between the second law and least action. Haller's identity establishes that the extremization of action *is* the extremization of entropy for a Bernoulli-Gaussian particle. This link, which in the broader physics literature remains a conjecture or a philosophical observation, is rendered as a theorem within Haller's model. ToE elevates this theorem from a single-particle result to a universal principle — but the single-particle theorem provides the proof of concept.

9.3 The Above Relation Stated Here in a More Precise Fashion

> **"Haller's paper is mathematically adjacent to the Theory of Entropicity and serves as a single-particle phenomenological sector of the broader entropic field theory. The Theory of Entropicity (ToE) can absorb Haller's result as a limiting case — the worldline-localized, non-relativistic, Gaussian-channel, $\alpha = 0$ sector of the full entropic dynamics — while extending the entropic**

> **principle to the full geometric, dynamical, and ontological structure of the universe. The Haller-Obidi Action and Lagrangian are the named mathematical objects that mediate this absorption: they translate between Haller's information-theoretic particle identity and ToE's covariant field formulation, preserving the mathematical content of both while revealing the structural hierarchy that connects them."**

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10. Conclusion

This Letter (***Letter IB*** in the **Theory of Entropicity (ToE) Living Review Letters Series**) has established the precise mathematical and structural connections between John L. Haller Jr.'s 2015 entropy-action identity and the entropic field action formulation of the Theory of Entropicity (ToE). The analysis has introduced several new mathematical constructions, each serving a specific function in the bridge between particle-level information theory and field-level entropic dynamics:

1. The Haller-Obidi Lagrangian $\mathcal{L}_{\mathrm{HO}} = mc^2 - (\hbar/2)\dot{H}$ (Definition 3.1, equation (13)) — the explicit entropic Lagrangian at the single-particle level, obtained by rearranging Haller's entropy-rate identity into a variational form. This Lagrangian admits Euler-Lagrange treatment and encodes the dual mechanical-informational character of classical trajectories.

2. The Haller-Obidi Action $S_{\mathrm{HO}} = \int \mathcal{L}_{\mathrm{HO}}\, dt = S_{\mathrm{action}}$ (Definition 3.2, equations (14)–(17)) — the time integral of the Haller-Obidi Lagrangian, shown to be identically equal to the classical action. The classical action is the entropic action, expressed in information-theoretic variables. This identity is exact within Haller's non-relativistic framework.

3. The covariant Haller-Obidi Lagrangian $\mathcal{L}_{\mathrm{ent}} = mc^2 - (\hbar/2)(u^\mu\, \partial_\mu\, S)$ (Definition 4.1, equation (24)) — the generally covariant extension of the Haller-Obidi Lagrangian, coupling particle motion (via four-velocity u^μ) to the ambient entropic field (via $\partial_\mu\, S$). This Lagrangian lives on arbitrary pseudo-Riemannian manifolds and reduces to $\mathcal{L}_{\mathrm{HO}}$ in the non-relativistic limit.

4. The Obidi-Haller Correspondence (Proposition 5.1, equations (38)–(40)) — the formal demonstration that the Obidi Action reduces to the covariant Haller-Obidi Action upon worldline localization of the entropic field. This establishes the hierarchy: Obidi Action (field level) → Haller-Obidi Action (worldline level) → Classical Action (non-relativistic limit), with each level emerging from the one above by mathematical restriction.

5. The information-geometric bridge (Section 6, equation (44)) — the identification of Haller's mutual information rate with an effective Fisher-Rao metric on the space of vacuum configurations, providing a constructive route from mutual information to the emergent physical metric of ToE. The entropic coupling constants g_{ent} (equation (42)) parameterize the informational strength of fundamental interactions.

6. The Vuli-Ndlela bridge (Section 8, equations (48)–(49)) — the connection between Haller's entropy-weighted path selection and the Vuli-Ndlela path integral of ToE. Haller's result demonstrates that the standard Feynman path integral already contains an implicit entropy weighting; the Vuli-Ndlela Integral makes this explicit and generalizes it through the entropic selection parameter λ, introducing an intrinsic time asymmetry that the standard formulation lacks.

These six constructions demonstrate that Haller's entropy-action identity and the Obidi entropic field theory are not merely analogous or philosophically aligned — they are **mathematically nested**. The Haller-Obidi Action is the single-particle projection of the Obidi Action. The classical Lagrangian is the informational residue of the entropic field evaluated along a worldline. The information-theoretic decomposition $H = H_C + I_M$ is the single-particle projection of the field-theoretic decomposition $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$. The mutual information rate is the single-particle value of the Fisher-Rao metric. At every level of description, the particle-level information theory and the field-level entropic dynamics map onto each other through precisely defined mathematical correspondences.

The limits of the correspondence are equally precise. Haller does not construct an entropic field, conserved flux, field equations, or intrinsic time asymmetry — these are ToE structures that emerge only at the field-theoretic level. But within his domain of validity (single particle, non-relativistic, Gaussian channel, $\alpha = 0$ sector), Haller's results are exact and provide an independently derived confirmation of ToE's central claim: that entropy and action are two faces of the same mathematical structure.

Several directions for future work emerge naturally from this analysis. First, the **many-body generalization** of the Haller-Obidi Lagrangian — the extension from a single particle to a system of interacting particles, with mutual information between all pairs — would provide the first concrete multi-particle sector of the Obidi Action. Second, the **fully relativistic extension** of Haller's derivation — replacing the Taylor expansions in v/c with exact Lorentz-covariant expressions — would establish the Haller-Obidi correspondence at all velocities. Third, the **explicit derivation** of the Haller-Obidi Action from the Obidi Field Equations — constructing the localized solution $S(x)$ that, upon worldline restriction, yields exactly the Haller-Obidi Lagrangian — would elevate the Obidi-Haller Correspondence from an ansatz-dependent result to a theorem of the Obidi Field Equations. Fourth, the **experimental signatures** of the entropic selection parameter λ — the correlations between irreversibility and transition-amplitude deviations predicted by the Vuli-Ndlela Integral — offer a concrete avenue for empirical tests of the entropic selection principle.

The Haller-Obidi Action and Lagrangian, defined and analyzed in this Letter, serve as the mathematical hinge between information-theoretic particle mechanics and the full entropic field theory of the Theory of Entropicity (ToE). They show that the relation between entropy and action—first recognized as a philosophical alignment and later established as a formal identity by Haller—constitutes a genuine structural theorem with precise mathematical content, well-defined limits, and generative consequences for the foundations and formulation of the Theory of Entropicity (ToE).

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