FOUNDATIONS OF THE THEORY OF ENTROPICITY (TOE) · OBIDI ACTION PRINCIPLE · INFORMATION GEOMETRY · ENTROPIC FIELD THEORY · MAY 18, 2026

# Foundations of the *Theory of Entropicity (ToE)* : The Obidi Action Principle (OAP) and the Geometry of the Entropic Field from Information Geometry (IG)

A FORMAL MONOGRAPH IN THE THEORY OF ENTROPICITY (TOE), INFORMATION GEOMETRY, ENTROPIC FIELD THEORY, AND ONTODYNAMICS

ABSTRACT

## Overview of the Obidi Action Principle

This monograph develops the full mathematical and conceptual foundations of the **Obidi Action Principle (OAP)**, the central variational structure underlying the **Theory of Entropicity (ToE)**. The OAP unifies three geometric sectors— Fisher–Rao, Fubini–Study, and Amari–Čencov $\alpha$-connections—into a single entropic manifold

whose curvature, metric, and affine structure jointly determine the dynamics of spacetime, matter, and gauge interactions.

Unlike traditional physics, where geometry, matter, and gauge fields are introduced as separate entities, ToE derives all three from the **information-geometric structure** of the entropic field. The Fisher–Rao metric becomes the spacetime metric; the Fubini–Study metric becomes the matter–energy sector; and the $\alpha$-connections become the gauge sector.

The **Obidi Action** is the unique diffeomorphism-invariant scalar functional constructed from these structures, and its variation yields the full field equations of ToE: a generalized **Einstein–Obidi field equation (EOFE),** entropic matter equations, and entropic gauge equations. This monograph presents the derivation, mathematical justification, and physical interpretation of these results in a rigorous and unified framework.

New to this work is the recognition that **information geometry is not merely a statistical or epistemic tool but the ontological substrate of physical reality**. The entropic manifold $\mathcal{M}$ functions as a pre-geometric arena from which spacetime, quantum matter, and gauge interactions *emerge* as geometric projections of a single entropic field. In this view, the Fisher–Rao, Fubini–Study, and $\alpha$-connection sectors are not analogies to physical structures —they **are** the physical structures, made dynamical through the Obidi Action.

The OAP therefore provides a **unified geometric mechanism** for the emergence of Einsteinian gravity (EG), quantum-mechanical inertial structure, and Yang–Mills gauge fields. It also introduces a new conceptual layer—**ontodynamics**—in which the laws of physics arise from the geometry of being itself rather than from externally imposed dynamical postulates. This establishes the Theory of Entropicity as a candidate framework for a fully unified description of spacetime, matter, and interaction fields, grounded in the mathematics of information geometry and the variational calculus of the entropic field.

# Introduction: The Entropic Manifold and the Ontological Elevation of Information Geometry

The Theory of Entropicity (ToE) begins from a radical but mathematically precise premise: **information geometry is not epistemic but ontological**. In classical statistics, the Fisher–Rao metric measures distinguishability between probability distributions; in quantum mechanics, the Fubini–Study metric measures distinguishability between quantum states; and in information geometry, the $\alpha$-connections encode dualistic affine structures. Traditionally, these structures quantify *knowledge* about systems. In ToE, they quantify *being* itself.

The entropic manifold $\mathcal{M}$ is defined as the space of physically realized entropic configurations. Each point $\theta \in \mathcal{M}$ corresponds to a complete specification of the entropic field $S(x)$ , and the geometry of $\mathcal{M}$ encodes the physical laws governing the universe.

The Fisher–Rao metric $g_{ij}(\theta)$ defines the spacetime interval; the Fubini–Study metric $h_{AB}(\theta)$ defines the inertial and energetic structure of matter; and the $\alpha$-connections $\nabla^{(\alpha)}$ define the gauge structure. These three components form the **entropic trinity** of ToE.

The **Obidi Action Principle (OAP)** is the unified variational principle that governs the dynamics of this entropic manifold. It is constructed from scalar invariants of $g_{ij}$, $h_{AB}$, and $\nabla^{(\alpha)}$, and its variation yields the full field equations of ToE. The OAP is therefore the entropic analogue of the **Einstein–Hilbert Action (EHA)**, the Yang–Mills action, and the matter action, but unified into a single geometric functional.

**New to this work is the recognition that the structures of information geometry are not mathematical analogues of physical geometry but the very substrate from which physical geometry emerges.** The entropic manifold is not a model of spacetime; it is the pre-geometric arena from which spacetime, matter, and gauge interactions arise as projections of a single entropic field. This ontological elevation transforms the Fisher–Rao, Fubini–Study, and $\alpha$-connection sectors from statistical constructs into fundamental geometric constituents of reality.

This shift resolves a long-standing conceptual tension in theoretical physics: the coexistence of three independent geometric frameworks—Riemannian geometry for gravity, Kähler geometry for quantum mechanics, and principal-bundle geometry for gauge fields. In ToE, these are not separate structures requiring unification; they are *different faces of the same entropic geometry*. Their unification is not imposed but inevitable, arising from the intrinsic geometry of $\mathcal{M}$.

The introduction of the entropic manifold therefore provides a natural explanation for the geometric unity of physical law. It shows that the curvature of spacetime, the inertial structure of quantum matter, and the holonomies of gauge fields are all manifestations of a single underlying principle: **the dynamics of entropy as geometry**. This monograph develops this principle in full mathematical detail and establishes the **Obidi Action** as the foundational variational structure governing the geometry of being.

§ II

## The Entropic Manifold $\mathcal{M}$: Coordinates, Structure, and Mathematical Foundations

The entropic manifold $\mathcal{M}$ is a smooth, finite- or infinite-dimensional differentiable manifold whose points represent entropic configurations of the universe. A coordinate chart $\theta = (\theta^1, \theta^2, \dots, \theta^n)$ labels these configurations. The probability distribution $p(x|\theta)$ is not epistemic but ontological: it is the local expression of the entropic field $S(x)$.

The manifold is equipped with a measure $\mu(x)$ such that $p(x|\theta)\, \mu(x)\, dx$ defines the entropic density. The Fisher–Rao metric is defined by:

$$g_{ij}(\theta) = \int p(x|\theta)\, \partial_i \log p(x|\theta)\, \partial_j \log p(x|\theta)\, dx.$$

The Čencov–Morozova theorem ensures that this metric is the **unique** Riemannian metric invariant under sufficient statistics and Markov morphisms. This uniqueness is the mathematical justification for identifying $g_{ij}$ with the spacetime metric: no other metric on $\mathcal{M}$ satisfies the required invariance properties.

The entropic manifold also carries an internal sector described by the Fubini–Study metric $h_{AB}(\theta)$, defined for pure states $|\psi(\lambda)\rangle$ by:

$$ds^2 = 4\left(\langle \partial_\lambda \psi \mid \partial_\lambda \psi \rangle - |\langle \psi \mid \partial_\lambda \psi \rangle|^2\right) d\lambda^2.$$

This metric defines a Kähler geometry whose curvature encodes the inertial and energetic structure of the entropic field. Finally, the manifold carries a family of $\alpha$-connections $\nabla^{(\alpha)}$, whose curvature defines the gauge field strengths.

Together, these structures define the full geometric content of the entropic manifold. The **Obidi Action** is constructed from these structures and governs their dynamics.

A central conceptual feature of the entropic manifold is that it is *pre-geometric*: the manifold $\mathcal{M}$ does not presuppose spacetime, but rather **spacetime is a derived structure** encoded in the Fisher–Rao metric. The coordinates $\theta^i$ are therefore not spacetime coordinates in the conventional sense; they are *entropic coordinates* whose geometric relations give rise to the familiar spatiotemporal relations only after the metric sector becomes dynamical through the Obidi Action.

Although the Fisher–Rao metric is often expressed in coordinates, its most fundamental definition is coordinate-free. For any tangent vector $V \in T_\theta \mathcal{M}$, the metric satisfies:

$$g_\theta(V, V) = \mathbb{E}_{p(x|\theta)}\left[\left(V \log p(x|\theta)\right)^2\right],$$

where $V \log p$ denotes the directional derivative of $\log p$ along $V$. This formulation makes explicit that the geometry of $\mathcal{M}$ is determined by the **response of the entropic field to infini-**

**tesimal deformations**.

The family of $\alpha$-connections endows $\mathcal{M}$ with a dualistic affine structure. For each $\alpha$, the pair $(\nabla^{(\alpha)}, \nabla^{(-\alpha)})$ forms a dual connection pair with respect to the Fisher–Rao metric:

$$X\, g(Y, Z) = g(\nabla^{(\alpha)}_X Y, Z) + g(Y, \nabla^{(-\alpha)}_X Z).$$

This duality is a structural feature unique to information geometry and has no analogue in classical Riemannian geometry. In ToE, this duality becomes the mathematical origin of **charge conjugation and gauge dualities** in the physical sector.

The Fubini–Study metric belongs to a full Kähler triple $(h, J, \omega)$, where $J$ is the complex structure and $\omega$ is the symplectic form. These satisfy:

$$\omega(X, Y) = h(JX, Y), \qquad J^2 = -\mathrm{Id}.$$

This structure implies that the internal sector of the entropic manifold carries a **canonical symplectic geometry**, making the Fubini–Study sector the natural geometric origin of *quantum phase space*. In ToE, this is not an analogy: quantum mechanics emerges as the **Kähler-geometric shadow** of the entropic manifold.

The skewness tensor $T_{ijk}$, which appears in the definition of the $\alpha$-connections, plays a role analogous to torsion in affine geometry. In ToE, this tensor encodes **asymmetries in the entropic field** and becomes the geometric origin of chiral interactions and parity-violating gauge phenomena.

Because the entropic manifold is the ontological substrate of physical law, the dimensionful constants of physics—such as the speed of light, Planck's constant, and gauge couplings—arise as **ratios of geometric invariants** of $g_{ij}$, $h_{AB}$, and $\nabla^{(\alpha)}$. In this framework, physical constants are not fundamental inputs but *derived quantities* determined by the geometry of $\mathcal{M}$.

The Čencov–Morozova theorem does more than guarantee uniqueness: it implies that any metric on the space of entropic configurations that respects the natural morphisms of the theory must **reduce to the Fisher–Rao metric**. Thus, the identification of $g_{ij}$ with the spacetime metric is not a choice but a mathematical inevitability.

The assignment $\theta \mapsto p(x|\theta)$ defines a functor from the category of entropic configurations to the category of measurable spaces. The Fisher–Rao metric, Fubini–Study metric, and $\alpha$-connections arise as **natural transformations** of this functor. This categorical perspective clarifies why the geometric structures of ToE are *canonical* rather than arbitrary.

In ToE, the light-cone structure of spacetime emerges from the **level sets of statistical distinguishability**. Directions in $\mathcal{M}$ along which $p(x|\theta)$ changes minimally correspond to causal propagation, while directions of maximal change correspond to spacelike separation. Thus, *causality is an entropic phenomenon*.

§ III

# The Fisher–Rao Metric as the Spacetime Sector of the Entropic Manifold

The Fisher–Rao metric occupies a privileged position within the Theory of Entropicity (ToE). It is not merely a statistical measure of distinguishability; it is the **unique** Riemannian metric compatible with the transformation laws of information-preserving mappings. This uniqueness, guaranteed by the Čencov–Morozova theorem, elevates the Fisher–Rao metric from a statistical artifact to a **fundamental geometric structure** of the entropic manifold. In ToE, this structure is identified with the **spacetime metric**.

Given a family of entropic configurations $p(x|\theta)$, the Fisher–Rao metric is:

$$g_{ij}(\theta) = \int p(x|\theta)\, \partial_i \log p(x|\theta)\, \partial_j \log p(x|\theta)\, dx.$$

This metric measures the infinitesimal distinguishability between neighboring entropic states $\theta$ and $\theta + d\theta$. In classical information geometry, this is interpreted epistemically: it quantifies how well an observer can discriminate between two statistical models. In ToE, the interpretation is ontological: the distinguishability is a **physical** separation between two possible configurations of the entropic field $S(x)$.

The Čencov–Morozova theorem states that the Fisher–Rao metric is the only Riemannian metric invariant under sufficient statistics and Markov morphisms. In the entropic ontology, these invariances correspond to the requirement that physical laws be invariant under coarse-graining and entropic transformations. Thus, the Fisher–Rao metric is the **only** metric compatible with the fundamental symmetries of the entropic field.

The geodesics of the Fisher–Rao metric satisfy:

$$\frac{d^2\theta^i}{d\tau^2} + \Gamma^i{}_{jk}(\theta)\,\frac{d\theta^j}{d\tau}\,\frac{d\theta^k}{d\tau} = 0,$$

where $\Gamma^i{}_{jk}$ are the Christoffel symbols of $g_{ij}$. These geodesics represent the **physically realized trajectories** of the entropic field. The entropic interval:

$$ds^2 = g_{ij}(\theta)\,d\theta^i d\theta^j$$

is the analogue of the spacetime interval in general relativity. The manifold $(\mathcal{M}, g_{ij})$ is therefore the **spacetime sector** of ToE.

DIAGRAM: FISHER–RAO AS SPACETIME

Entropic Configurations $\theta \longrightarrow$ Fisher–Rao Metric $g_{ij} \longrightarrow$ Spacetime Geometry

Uniqueness (Čencov–Morozova) $\Rightarrow$ Physical Necessity

A deeper understanding of the Fisher–Rao metric arises from its coordinate-free definition. For any tangent vector $V \in T_\theta \mathcal{M}$, the metric satisfies:

$$g_\theta(V, V) = \mathbb{E}_{p(x|\theta)}\left[(V \log p(x|\theta))^2\right].$$

This expression shows that the Fisher–Rao metric measures the **second-order sensitivity** of the entropic field to infinitesimal deformations. In ToE, this sensitivity is interpreted physically: directions in which $p(x|\theta)$ changes minimally correspond to *causal propagation*, while directions of maximal change correspond to *spacelike separation*. Thus, the causal structure of spacetime emerges from **statistical distinguishability**.

The Fisher–Rao metric also admits a natural **pseudo-Riemannian extension** when the entropic field satisfies global normalization and conservation constraints. These constraints induce a decomposition of the tangent space into directions of positive, negative, and null distinguishability, giving rise to an emergent **Lorentzian signature**. In this sense, the light-cone structure of spacetime is a direct consequence of the entropic geometry of $\mathcal{M}$.

The Christoffel symbols $\Gamma^i{}_{jk}$ of the Fisher–Rao metric acquire a physical interpretation in ToE. They encode the **entropic inertia** of the field: the resistance of the entropic configuration to changes in its statistical structure. The geodesic equation

$$\frac{d^2\theta^i}{d\tau^2} + \Gamma^i{}_{jk}(\theta)\,\frac{d\theta^j}{d\tau}\,\frac{d\theta^k}{d\tau} = 0$$

is therefore not merely a geometric identity but the **equation of motion** for the entropic field. A freely evolving entropic configuration follows a path of extremal statistical length, which becomes the physical trajectory of a system in spacetime.

The curvature tensor $R^i{}_{jkl}$ of the Fisher–Rao metric measures the non-commutativity of successive entropic deformations. In ToE, this curvature is interpreted as the **gravitational field**. The Ricci tensor $R_{ij}$ and scalar curvature $R$ quantify the degree to which the entropic manifold deviates from statistical flatness. Thus, gravitational dynamics arise from the **curvature of statistical distinguishability**.

An important consequence of the Čencov–Morozova theorem is that the Fisher–Rao metric is not only unique but *functorial*: any morphism between entropic systems induces a metric-preserving map between their corresponding manifolds. This functoriality ensures that the spacetime sector of ToE is **canonical** and independent of arbitrary choices of parametrization or representation.

Finally, the entropic interval

$$ds^2 = g_{ij}(\theta)\, d\theta^i d\theta^j$$

acquires a physical interpretation as the **minimal entropic cost** required to move between two configurations of the universe. In this sense, spacetime distance is not a primitive notion but a *derived measure of entropic change*. The Fisher–Rao metric therefore provides the mathematical foundation for the spacetime sector of the Theory of Entropicity.

§ IV

## The Fubini–Study Metric as the Matter–Energy Sector of the Entropic Manifold

The Fubini–Study metric arises naturally in the geometry of quantum states, but in the Theory of Entropicity it is reinterpreted as the **matter–energy sector** of the entropic manifold. This reinterpretation is not metaphorical; it is a mathematically rigorous identification of curvature in the internal sector of the entropic manifold with the inertial and energetic properties of physical systems.

For pure states $|\psi(\lambda)\rangle$, the Fubini–Study metric is:

$$ds^2 = 4\left(\langle \partial_\lambda \psi \mid \partial_\lambda \psi \rangle - |\langle \psi \mid \partial_\lambda \psi \rangle|^2\right) d\lambda^2.$$

This metric defines a Kähler geometry on the projective Hilbert space. Its curvature tensor $R_{ABCD}[h]$ encodes the "stiffness" of the entropic field along internal directions. In ToE, this stiffness is interpreted as **inertial resistance**.

Directions in the entropic manifold that are costly in the Fubini–Study metric correspond to directions in which the entropic configuration resists deformation. This resistance is the entropic analogue of mass.

The scalar curvature $K[h]$ of the Fubini–Study sector contributes to the Obidi Action via:

$$L_{\mathrm{FS}}[h, g] = -\frac{1}{2}\,\mu_{\mathrm{FS}}\,\mathrm{Tr}_g(K[h]),$$

where $\mu_{\mathrm{FS}}$ is a coupling constant and $\mathrm{Tr}_g$ denotes an appropriate contraction of internal curvature with the spacetime metric.

Regions where $K[h] \neq 0$ correspond to regions where the entropic field stores energy, mass, or internal degrees of freedom. A flat Fubini–Study geometry corresponds to vacuum-like behavior. Thus, matter and energy are not external inputs to the theory; they are **curvature phenomena** of the entropic manifold.

DIAGRAM: FUBINI-STUDY AS MATTER-ENERGY

Internal Geometry $h_{AB}$ $\longrightarrow$ Curvature $K[h]$ $\longrightarrow$ Matter/Energy Content

Matter = Curvature of the Entropic Field

A deeper understanding of the Fubini–Study sector begins with its full Kähler structure. The metric $h_{AB}$, complex structure $J^A{}_B$, and symplectic form $\omega_{AB}$ satisfy:

$$h_{AB} = \omega_{AC} J^C{}_B, \qquad J^A{}_C J^C{}_B = -\delta^A{}_B.$$

This triple endows the internal sector of the entropic manifold with a **canonical complex-symplectic geometry**. In ToE, this structure is not an import from quantum mechanics; rather, quantum mechanics emerges as the *holomorphic shadow* of the entropic manifold. The complex structure $J$ encodes phase, the symplectic form $\omega$ encodes interference, and the metric $h$ encodes inertial response.

The curvature tensor $R_{ABCD}[h]$ of the Fubini–Study metric measures the non-commutativity of internal deformations of the entropic field. In ToE, this curvature is interpreted as **internal stress** or **entropic inertia**. Directions in which the curvature is large correspond to internal degrees of freedom that resist deformation, giving rise to the physical manifestation of *mass*.

The scalar curvature $K[h]$ plays a central role in the Obidi Action. Its contribution

$$L_{\text{FS}}[h, g] = -\frac{1}{2}\,\mu_{\text{FS}}\,\text{Tr}_g(K[h])$$

shows that the internal curvature couples directly to the spacetime metric $g_{ij}$. This coupling is the geometric origin of the **mass–energy relation** in ToE: the energetic content of a configuration is proportional to the curvature of its internal geometry.

The Fubini–Study sector also encodes the **geometry of quantum superposition**. Given two internal directions $X$ and $Y$, the interference term

$$\omega(X, Y)$$

measures the symplectic area swept out by their combined evolution. In ToE, this area is interpreted as the *entropic phase* accumulated by the internal configuration. Thus, quantum interference is not a mysterious phenomenon but a **geometric effect** of the internal symplectic structure of $\mathcal{M}$.

The geodesics of the Fubini–Study metric satisfy:

$$\frac{D^2\phi^A}{d\tau^2} + \Gamma^A{}_{BC}[h]\,\frac{d\phi^B}{d\tau}\,\frac{d\phi^C}{d\tau} = 0,$$

where $\phi^A$ are internal coordinates. These geodesics represent the **natural evolution** of internal degrees of freedom in the absence of external entropic forces. In the emergent quantum-mechanical limit, these geodesics reduce to Schrödinger evolution on projective Hilbert space.

An important structural feature of the Fubini–Study sector is its **constant holomorphic sectional curvature**. This property ensures that the internal geometry is maximally symmetric, making it the natural candidate for the internal sector of a unified theory. In ToE, deviations from constant curvature correspond to *excited internal states*, which manifest physically as particles, fields, and matter content.

Finally, the Fubini–Study metric provides a natural measure of **internal distance** between entropic configurations. Two configurations that differ only by a global phase are identified, while configurations that differ by internal structure are separated by a finite entropic distance. This distance is the geometric origin of *quantum state distinguishability* and, in ToE, becomes the measure of **internal energetic separation**.

§ V

## The $\alpha$-Connections as the Gauge Sector of the Entropic Manifold

The third geometric component of the entropic manifold is the family of Amari–Čencov $\alpha$-connections. In classical information geometry, these connections encode dualistic affine structures and determine how statistical quantities are parallel transported. In the Theory of Entropicity, they are elevated to the status of **gauge fields**.

On the statistical manifold $\mathcal{M}$ with Fisher–Rao metric $g_{ij}$, the $\alpha$-connections $\nabla^{(\alpha)}$ are defined by:

$$\Gamma^{(\alpha)}_{ijk} = \Gamma^{(0)}_{ijk} + \frac{\alpha}{2} T_{ijk},$$

where $\Gamma^{(0)}_{ijk}$ is the Levi–Civita connection of $g_{ij}$, and $T_{ijk}$ is the skewness tensor:

$$T_{ijk} = \int p(x|\theta)\, \partial_i \partial_j \partial_k \log p(x|\theta)\, dx.$$

In a gauge-theoretic representation, one introduces gauge potentials $A_i^{(\alpha)\,a}$ valued in a Lie algebra with structure constants $f^a{}_{bc}$. The curvature (field strength) of the $\alpha$-connection is:

$$F_{ij}^{(\alpha)\,a} = \partial_i A_j^{(\alpha)\,a} - \partial_j A_i^{(\alpha)\,a} + f^a{}_{bc}\, A_i^{(\alpha)\,b}\, A_j^{(\alpha)\,c}.$$

In ToE, the $\alpha$-connections define how internal degrees of freedom are parallel transported along the entropic manifold. This is precisely the role played by gauge fields in Yang–Mills theory. The curvature $F_{ij}^{(\alpha)\,a}$ is interpreted as the **gauge field strength**, and the holonomies of $\nabla^{(\alpha)}$ correspond to physical charges and phase factors.

The gauge sector contributes to the **Obidi Action** through:

$$L_{\text{gauge}}[F^{(\alpha)}, g] = -\frac{1}{4} \sum_{\alpha} \kappa_\alpha\, g^{ik} g^{jl}\, \eta_{ab}\, F_{ij}^{(\alpha)\,a}\, F_{kl}^{(\alpha)\,b}.$$

Thus, gauge fields are not added to the theory as external structures; they are **intrinsic affine structures** of the entropic manifold. The $\alpha$-connections complete the entropic trinity: Fisher–Rao gives spacetime, Fubini–Study gives matter, and $\alpha$-connections give gauge interactions.

DIAGRAM: A-CONNECTIONS AS GAUGE FIELDS

Affine Structure $\nabla^{(\alpha)} \longrightarrow$ Curvature $F^{(\alpha)} \longrightarrow$ Gauge Interactions

Gauge Fields = Affine Geometry of the Entropic Field

# Construction of the Obidi Action on the Entropic Manifold

**The Obidi Action Principle (OAP) is the central variational structure of the Theory of Entropicity (ToE). It unifies the three geometric sectors of the entropic manifold— Fisher–Rao, Fubini–Study, and Amari–Čencov $\alpha$-connections—into a single diffeomorphism-invariant scalar functional.** The OAP is not an arbitrary construction; it is the unique action compatible with the symmetries, invariances, and ontological commitments of the entropic field.

Let $\mathcal{M}$ be the entropic manifold with coordinates $\theta^i$. The geometric data on $\mathcal{M}$ consist of:

1. The Fisher–Rao metric $g_{ij}(\theta)$, defining the spacetime sector.
2. The Fubini–Study metric $h_{AB}(\theta)$, defining the matter–energy sector.
3. The $\alpha$-connections $\nabla^{(\alpha)}$, defining the gauge sector.

The action must satisfy three fundamental requirements:

i. Diffeomorphism invariance on $\mathcal{M}$.
ii. Construction from scalar contractions of curvature and metric tensors.
iii. Reduction to known physical actions **(Einstein–Hilbert, Yang–Mills, matter actions)** in appropriate limits.

Let $R[g]$ denote the **Ricci scalar of the Fisher–Rao metric.** Let $K[h]$ denote the scalar curvature of the Fubini–Study metric. Let $F_{ij}^{(\alpha)\,a}$ denote the curvature (field strength) of the $\alpha$-connection. Then, the celebrated **Obidi Action** is given compactly as follows:

$$S_{\text{Obidi}} = \int_{\mathcal{M}} d^n\theta \sqrt{|g|} \left[ \frac{1}{16\pi G_E} R[g] + L_{\text{FS}}[h,g] + L_{\text{gauge}}[F^{(\alpha)}, g] + L_{\text{int}}[g,h,F^{(\alpha)}] \right].$$

Here $n$ is the dimension of the entropic manifold, and $G_E$ is the entropic gravitational constant. The **Fubini–Study (matter–energy) sector** contributes:

$$L_{\text{FS}}[h, g] = -\frac{1}{2}\,\mu_{\text{FS}}\,\mathrm{Tr}_g(K[h]),$$

where $\mu_{\text{FS}}$ is a coupling constant and $\mathrm{Tr}_g$ denotes an appropriate contraction of internal curvature with the spacetime metric.

The **gauge sector contributes**:

$$L_{\text{gauge}}[F^{(\alpha)}, g] = -\frac{1}{4}\sum_{\alpha}\kappa_\alpha\, g^{ik} g^{jl}\,\eta_{ab}\,F_{ij}^{(\alpha)\,a}\,F_{kl}^{(\alpha)\,b}.$$

The interaction term $L_{\text{int}}$ contains all non-minimal couplings between the three sectors, constrained by **diffeomorphism invariance, gauge invariance, and entropic symmetry.**

DIAGRAM: STRUCTURE OF THE OBIDI ACTION

Spacetime (Fisher–Rao) + Matter (Fubini–Study) + Gauge ($\alpha$-Connections)

$\longrightarrow$ Unified Variational Principle $S_{\text{Obidi}}$

§ VII

## Variational Calculus on the Entropic Manifold

To derive the field equations of the Theory of Entropicity, one performs a systematic variation of the Obidi Action with respect to its independent geometric variables: the Fisher–Rao metric $g_{ij}$, the Fubini–Study metric $h_{AB}$ (or equivalently the internal fields $\phi^A$), and the gauge potentials $A_i^{(\alpha)\,a}$ associated with the $\alpha$-connections. Each variation yields a distinct sector of the full ToE field equations.

### *VII.1 Variation with Respect to the Fisher–Rao Metric*

Writing the action as:

$$S_{\text{Obidi}} = \int d^n\theta \sqrt{|g|}\, L_{\text{total}},$$

with:

$$L_{\text{total}} = \frac{1}{16\pi G_E} R[g] + L_{\text{FS}} + L_{\text{gauge}} + L_{\text{int}},$$

the metric variation yields:

$$\delta S_{\text{Obidi}} = \int d^n\theta \sqrt{|g|} \left[ \frac{1}{16\pi G_E} (G_{ij} + \Lambda_{ij}) - \frac{1}{2} T_{ij}^{(\text{total})} \right] \delta g^{ij}.$$

Setting $\delta S_{\text{Obidi}} = 0$ for arbitrary $\delta g^{ij}$ gives the generalized **Einstein–Obidi field equations (EOFE)**:

$$G_{ij} + \Lambda_{ij} = 8\pi G_E T_{ij}^{(\text{total})}.$$

Here $\Lambda_{ij}$ contains non-minimal couplings, and $T_{ij}^{(\text{total})}$ is the total entropic stress–energy tensor.

### *VII.2 Variation with Respect to the Fubini–Study Sector*

Let $\phi^A(\theta)$ parametrize the internal manifold. The internal metric $h_{AB}(\phi)$ and its curvature $K[h]$ become functionals of $\phi^A$. The variation yields:

$$\frac{\delta S_{\text{Obidi}}}{\delta \phi^A} = 0 \quad \Longrightarrow \quad E_A[\phi, g, F^{(\alpha)}] = 0.$$

These equations describe the dynamics of the **entropic matter sector, including mass generation, self-interaction, and coupling to gauge and gravitational fields.**

### *VII.3 Variation with Respect to the α-Connections*

Let $A_i^{(\alpha)a}$ be the gauge potentials whose curvature is $F_{ij}^{(\alpha)a}$. The variation yields:

$$\frac{\delta S_{\text{Obidi}}}{\delta A_i^{(\alpha)}{}_a} = 0 \quad \Longrightarrow \quad D_j\left(\sqrt{|g|}\, g^{jk} g^{il}\, \eta_{ab}\, F_{kl}^{(\alpha)b}\right) = \sqrt{|g|}\, J^{(\alpha)i}{}_a.$$

Here $D_j$ is the **gauge-covariant derivative,** and $J^{(\alpha)i}{}_a$ is the **entropic gauge current arising from the internal sector and interaction terms.**

These are the entropic generalizations of the Yang–Mills equations.

DIAGRAM: VARIATIONAL STRUCTURE OF TOE

$\delta S/\delta g_{ij} \longrightarrow$ Gravitational Sector

$\delta S/\delta \phi^A \longrightarrow$ Matter Sector

$\delta S/\delta A_i^{(\alpha)} \longrightarrow$ Gauge Sector

All from a Single Action $S_{\text{Obidi}}$

§ VIII

## The Full Field Equations of the Theory of Entropicity (ToE)

**Thus, we have seen, from all of the above, that the field equations of the Theory of Entropicity (ToE) arise from the simultaneous variation of the Obidi Action with respect to all geometric variables.** These equations are not independent; **they**

**are three projections of a single variational principle on the entropic manifold.** Together, **they form a unified system describing the dynamics of spacetime, matter, and gauge fields.**

### *VIII.1 Generalized Einstein–Obidi Field Equations (GEOFE)*

$$G_{ij} + \Lambda_{ij} = 8\pi G_E T_{ij}^{(\text{total})}.$$

This equation **generalizes Einstein's field equations (EFE).** The **Fisher–Rao metric** $g_{ij}$ **plays the role of the spacetime metric,** and the **total entropic stress–energy tensor includes contributions from the Fubini–Study sector, the gauge sector, and their interactions.**

### *VIII.2 Entropic Matter Equations*

$$E_A[\phi, g, F^{(\alpha)}] = 0.$$

These **equations describe the dynamics of the internal entropic fields.** They **include mass terms, self-interaction terms, and couplings to gauge and gravitational fields.** They are the **entropic analogue of the Klein–Gordon, Dirac, or nonlinear sigma-model equations,** depending on the structure of $h_{AB}$.

### *VIII.3 Entropic Gauge Equations*

$$D_j\left(\sqrt{|g|}\, g^{jk} g^{il}\, \eta_{ab}\, F_{kl}^{(\alpha)\,b}\right) = \sqrt{|g|}\, J^{(\alpha)i}{}_{a}.$$

These are the **entropic generalizations of the Yang–Mills equations.** The $\alpha$-connections define the gauge structure, and their curvature defines the gauge field strengths. The currents $J^{(\alpha)i}{}_{a}$ arise from the internal entropic geometry.

**Together, these three sets of equations form the complete dynamical system of the Theory of Entropicity (ToE).** They **describe how the entropic field evolves, how space-**

**time curves, how matter behaves, and how gauge fields propagate—all as manifestations of a single geometric structure.**

DIAGRAM: THE FULL TOE FIELD SYSTEM

Generalized Einstein–Obidi Field Equations (GEOFE)

Entropic Matter Equations

Entropic Gauge Equations

Unified by the Obidi Action Principle

APPENDIX A

# Mathematical Preliminaries for the Entropic Manifold

This appendix gathers the mathematical structures required for the rigorous formulation of the Theory of Entropicity (ToE). The entropic manifold $\mathcal{M}$ is a differentiable manifold equipped with three geometric sectors: the Fisher–Rao metric $g_{ij}$, the Fubini–Study metric $h_{AB}$, and the Amari–Čencov $\alpha$-connections $\nabla^{(\alpha)}$. Each of these structures requires a precise mathematical foundation.

## *A.1 Differential Geometry of the Entropic Manifold*

Let $\mathcal{M}$ be an $n$-dimensional smooth manifold with coordinates $\theta^i$. A metric tensor $g_{ij}(\theta)$ defines the line element:

$$ds^2 = g_{ij}(\theta)\, d\theta^i d\theta^j.$$

The Levi–Civita connection associated with $g_{ij}$ has Christoffel symbols:

$$\Gamma^i{}_{jk} = \frac{1}{2} g^{i\ell} \left( \partial_j g_{\ell k} + \partial_k g_{\ell j} - \partial_\ell g_{jk} \right).$$

The Riemann curvature tensor is:

$$R^i{}_{jkl} = \partial_k \Gamma^i{}_{jl} - \partial_l \Gamma^i{}_{jk} + \Gamma^i{}_{km} \Gamma^m{}_{jl} - \Gamma^i{}_{lm} \Gamma^m{}_{jk}.$$

The Ricci tensor and scalar curvature follow:

$$R_{jl} = R^i{}_{jil}, \qquad R = g^{jl} R_{jl}.$$

## *A.2 Fisher–Rao Geometry*

Given a family of entropic configurations $p(x|\theta)$, the Fisher–Rao metric is:

$$g_{ij}(\theta) = \int p(x|\theta)\, \partial_i \log p(x|\theta)\, \partial_j \log p(x|\theta)\, dx.$$

The Čencov–Morozova theorem states that the Fisher–Rao metric is the *unique* Riemannian metric invariant under Markov morphisms. This uniqueness is the mathematical justification for identifying $g_{ij}$ with the spacetime metric in ToE.

## *A.3 Fubini–Study Geometry*

For pure states $|\psi(\lambda)\rangle$, the Fubini–Study metric is:

$$ds^2 = 4 \left( \langle \partial_\lambda \psi \mid \partial_\lambda \psi \rangle - |\langle \psi \mid \partial_\lambda \psi \rangle|^2 \right) d\lambda^2.$$

The Fubini–Study metric defines a Kähler manifold with complex structure $J$, symplectic form $\omega$, and metric $h$ satisfying:

$$h(JX, JY) = h(X, Y), \qquad \omega(X, Y) = h(JX, Y).$$

### A.4 Amari–Čencov α-Connections

The $\alpha$-connections are defined by:

$$\Gamma^{(\alpha)}_{ijk} = \Gamma^{(0)}_{ijk} + \frac{\alpha}{2} T_{ijk},$$

where the skewness tensor $T_{ijk}$ is:

$$T_{ijk} = \int p(x|\theta)\, \partial_i \partial_j \partial_k \log p(x|\theta)\, dx.$$

The curvature of the $\alpha$-connection is:

$$F^{(\alpha)\,a}_{ij} = \partial_i A^{(\alpha)\,a}_j - \partial_j A^{(\alpha)\,a}_i + f^a{}_{bc}\, A^{(\alpha)\,b}_i\, A^{(\alpha)\,c}_j.$$

APPENDIX B

# Worked Examples in Entropic Geometry

### B.1 Fisher–Rao Geometry of a Two-State System

Consider a binary distribution $p = (p, 1-p)$. The Fisher–Rao metric is:

$$g_{pp} = \frac{1}{p(1-p)}.$$

The line element is:

$$ds^2 = \frac{dp^2}{p(1-p)}.$$

The geodesic distance between $p_1$ and $p_2$ is:

$$d(p_1, p_2) = 2 \arccos\left( \sqrt{p_1 p_2} + \sqrt{(1-p_1)(1-p_2)} \right).$$

This is the well-known **Hellinger distance / Bhattacharyya angle** already derived in the standard formalism.

### *B.2 Fubini–Study Curvature of a Qubit*

A qubit state can be written as:

$$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle.$$

This is the standard **Bloch sphere representation.**

The Fubini–Study metric becomes:

$$ds^2 = \frac{1}{4}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$

which is the metric of a sphere of radius $1/2$ [this is the classical metric on $\mathbb{CP}^1$]. The scalar curvature is:

$$R = 8.$$

### *B.3 Holonomy of an α-Connection*

For a closed loop $\gamma$ in $\mathcal{M}$, the holonomy is:

$$U_\gamma^{(\alpha)} = \mathcal{P}\exp\left(\oint_\gamma A_i^{(\alpha)\,a}\, T_a\, d\theta^i\right),$$

where $\mathcal{P}$ denotes path ordering. This holonomy corresponds to a physical gauge phase in ToE.

# Scholium:

We must point this out here as elsewhere: in the above worked examples, ToE is original and radical [and audacious] not because it invents new metrics, but because it reinterprets Fisher–Rao, Fubini–Study, and $\alpha$-connections as the three physical sectors of reality and unifies them through a single entropic variational principle with radical physical consequences.

### *What is Original in the Theory of Entropicity?*

The originality of the **Theory of Entropicity (ToE)** does not lie in the classical mathematical objects it employs—such as the Fisher–Rao metric, the Fubini–Study metric, or the $\alpha$-connections. These structures are well established in information geometry, quantum geometry, and statistical theory. Rather, the originality of ToE lies in the *physical reinterpretation*, *ontological elevation*, and *unification* of these geometries into a single dynamical framework governed by the Obidi Action.

ToE makes a fundamentally new claim: that the Fisher–Rao metric constitutes the geometric sector of spacetime itself; that the Fubini–Study metric encodes the intrinsic curvature associated with matter–energy; and that the dualistic $\alpha$-connections provide the natural geometric origin of gauge fields and internal symmetries. These three sectors—traditionally treated as mathematically distinct—are shown to arise from a single underlying entropic manifold $\mathcal{M}$, whose geometry is not epistemic but ontological.

The **Obidi Action** is the central original contribution of the Theory of Entropicity (ToE). It unifies the Fisher–Rao, Fubini–Study, and $\alpha$-connection geometries into a single variational principle whose Euler–Lagrange equations yield the **Einstein–Obidi field equations (EOFE)**, the **entropic matter equations (EME)**, and the **entropic gauge equations (EGE)**. This establishes a coherent dynamical structure in which spacetime, matter, and interactions emerge from the same entropic field, rather than being introduced as separate postulates.

In this sense, the originality of ToE is philosophical, conceptual and structural: it provides a new ontological foundation for physics by treating entropy as the primitive field from which spacetime, matter, and gauge interactions emerge. The mathematical objects used in the construction are classical, but their *physical meaning*, *role*, and *unification* within the Obidi Action are entirely new. This reinterpretation [and mathematical restructuring] transforms information geometry from a descriptive framework into a fundamental physical geometry, thereby offering a unified entropic foundation for the laws of nature.

APPENDIX C

# Physical Limits of the Obidi Action

## *C.1 General Relativity Limit*

When the Fubini–Study and gauge sectors vanish, the **Obidi Action** reduces to:

$$S \to \frac{1}{16\pi G_E} \int d^n\theta \sqrt{|g|}\, R[g].$$

This is the **Einstein–Hilbert action** with gravitational constant $G_E$.

## *C.2 Yang–Mills Limit*

When $g_{ij}$ is fixed and $h_{AB}$ is trivial, the action reduces to:

$$S \to -\frac{1}{4} \sum_\alpha \kappa_\alpha \int d^n\theta \sqrt{|g|}\, g^{ik} g^{jl}\, \eta_{ab} F_{ij}^{(\alpha)\,a} F_{kl}^{(\alpha)\,b}.$$

*C.3 Quantum Mechanics as an Emergent Hilbert Chart*

When the entropic manifold admits a complex structure compatible with $h_{AB}$, the internal dynamics reduce to Schrödinger evolution in an emergent Hilbert space.

APPENDIX D

# Tables, Diagrams, and Conceptual Maps

DIAGRAM D.1 – THE ENTROPIC TRINITY

Fisher–Rao $\longrightarrow$ Spacetime

Fubini–Study $\longrightarrow$ Matter–Energy

$\alpha$-Connections $\longrightarrow$ Gauge Fields

Unified by the Obidi Action Principle

DIAGRAM D.2 – VARIATIONAL FLOW OF TOE

$\delta S/\delta g_{ij} \longrightarrow$ Gravitational Sector

$\delta S/\delta h_{AB} \longrightarrow$ Matter Sector

$\delta S/\delta A_i^{(\alpha)} \longrightarrow$ Gauge Sector

PART V

# Bibliography, Notation Index, and Glossary

## *V.1 Bibliography*

The following references provide the mathematical, physical, and conceptual background for the Theory of Entropicity (ToE), the Obidi Action Principle (OAP), and the information-geometric structures employed throughout this monograph.

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**[8]** M. Nakahara, *Geometry, Topology and Physics*, 2nd ed., Taylor & Francis, 2003.

**[9]** J. O. Obidi, *Foundational Letters on the Theory of Entropicity (ToE)*, internal monograph series (unpublished drafts, 2024–2026).

**[10]** J. O. Obidi, "The Obidi Action Principle (OAP) and the Entropic Manifold," working manuscript in entropic field theory, 2025–2026.

### *V.2 Notation Index*

This subsection collects the principal symbols used throughout the monograph. All indices follow the conventions stated below unless explicitly indicated otherwise.

$\mathcal{M}$ — Entropic manifold (space of entropic configurations).

$\theta^i$ — Coordinates on $\mathcal{M}$; spacetime-sector coordinates.

$x$ — Underlying micro-configuration variable.

$p(x|\theta)$ — Ontological probability distribution (entropic configuration).

$S(x)$ — Entropic field.

$g_{ij}$ — Fisher–Rao metric (spacetime sector).

$h_{AB}$ — Fubini–Study metric (matter–energy sector).

$\nabla^{(\alpha)}$ — Amari–Čencov $\alpha$-connection (gauge sector).

$\Gamma^i{}_{jk}$ — Christoffel symbols of $g_{ij}$.

$\Gamma^{(\alpha)}_{ijk}$ — Christoffel symbols of the $\alpha$-connection.

$T_{ijk}$ — Skewness tensor.

$R^i{}_{jkl}, R_{ij}, R$ — Riemann curvature, Ricci tensor, Ricci scalar.

$K[h]$ — Scalar curvature of the Fubini–Study sector.

$A_i^{(\alpha)\,a}$ — Gauge potential.

$F_{ij}^{(\alpha)\,a}$ — Gauge field strength.

$f^a{}_{bc}$ — Structure constants of the gauge algebra.

$\eta_{ab}$ — Invariant metric on the gauge algebra.

$G_{ij}$ — Einstein tensor.

$\Lambda_{ij}$ — Non-minimal coupling tensor.

$T_{ij}^{(\mathrm{total})}$ — Total entropic stress–energy tensor.

$\phi^A$ — Internal fields.

$E_A$ — Euler–Lagrange expressions for internal fields.

$J^{(\alpha)i}{}_a$ — Entropic gauge current.

$G_E$ — Entropic gravitational constant.

$\mu_{\mathrm{FS}}$ — Fubini–Study coupling constant.

$\kappa_\alpha$ — Gauge coupling constants.

$S_{\mathrm{Obidi}}$ — Obidi Action functional.

$L_{\mathrm{FS}}, L_{\mathrm{gauge}}, L_{\mathrm{int}}$ — Lagrangian densities.

## *V.3 Glossary of Core Concepts*

**Entropic Manifold ($\mathcal{M}$)** — The differentiable manifold whose points represent physically realized entropic configurations of the universe.

**Entropic Field ($S(x)$)** — A scalar or tensorial field representing local entropic density or configuration.

**Fisher–Rao Metric ($g_{ij}$)** — The unique information-geometric metric invariant under Markov morphisms; reinterpreted as the spacetime metric in ToE.

**Fubini–Study Metric ($h_{AB}$)** — The Kähler metric on projective Hilbert space; reinterpreted as the matter–energy sector.

**Amari–Čencov $\alpha$-Connections** — A one-parameter family of affine connections compatible with $g_{ij}$, reinterpreted as gauge connections.

**Obidi Action Principle (OAP)** — The unified variational principle governing the dynamics of the entropic manifold.

**Generalized Einstein–Obidi Field Equation (GEOFE)** — The gravitational field equation of ToE.

**Entropic Matter Equations** — Internal field equations governing the Fubini–Study sector.

**Entropic Gauge Equations** — Gauge field equations generalizing Yang–Mills theory.

**Ontodynamics** — The study of dynamical laws at the level of being itself.

**Hilbert Space as Emergent Chart** — The ToE view that Hilbert space is not fundamental but arises as a coordinate chart on certain sectors of the entropic manifold.

# PART VI

# Fully Linked Table of Contents

PART VII

# The ToE PDF-Ready Print Stylesheet

This ToE stylesheet ensures that this monograph prints cleanly and professionally when exported to PDF using the browser's "Print to PDF" function. It removes background textures, optimizes margins, and stabilizes page breaks for equations, tables, and diagrams.

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