**From Shannon Entropy to Spacetime: A Rigorous Derivation of the Obidi Action from Shannon Entropy via Information Geometry in the Theory of Entropicity (ToE)**

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**ToE Living Review Letters Series (ToE LRLS) — Letter IIF**
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Theory of Entropicity (ToE)
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**Abstract**
The Theory of Entropicity (ToE) elevates Shannon entropy from an epistemic measure to an ontological field whose dynamics underlie both quantum matter and classical spacetime. In this Letter we provide a rigorous, step-by-step derivation of the **Obidi Action**—the universal action of ToE—beginning with the Shannon entropy functional and the information-geometric structure it induces. Starting from the continuous Shannon entropy of a probability density on a differentiable manifold, we promote the density to a dynamical entropic field $\phi$ via the identification $p(x) = e^{-\phi(x)}$. The resulting scalar functional serves as the potential term of an action. The kinetic term is furnished by the Fisher information metric, the natural Riemannian metric of information geometry, expressed through the gradient of $\phi$. Varying the resulting proto-Obidi action yields the simplest form of the **Master Entropic Equation**, a nonlinear covariant field equation governing the pre-geometric dynamics of the entropic field. We then show how the inclusion of curvature invariants built from the Fisher–Rao metric—through the process of *curvature transfer*—completes the proto-action to the full Obidi Action, from which Einstein's equations of general relativity emerge as thermodynamic identities in a coarse-grained limit. The derivation establishes a direct logical chain from Shannon's original formula to the dynamics of spacetime, rooting gravitation and field theory in the geometry of distinguishability.

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## 1. Introduction

A deep conceptual divide has long separated information theory from fundamental physics. Shannon's entropy,

$$H = -\sum_i p_i \ln\, p_i,$$

quantifies uncertainty in a probability distribution, while the Einstein–Hilbert action,

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R,$$

encodes the dynamics of spacetime curvature. The Theory of Entropicity (ToE) [1, 2, 3] dissolves this divide by positing that **entropy is ontological**, not merely epistemic. In ToE, the probability distribution $p(x)$ over a configuration manifold is not a state of knowledge but a real physical field—the *entropic field* $\phi(x)$—whose variations constitute the fundamental substance of the universe.

From Shannon Entropy to Spacetime: A Rigorous Derivation of the Obidi Action from Shannon Entropy via Information Geometry in the Theory of Entropicity (ToE)

Previous Letters in this series have established the kinematical framework of ToE (Letter I), the Master Entropic Equation (Letter IIA), the No-Rush Theorem (Letter IIB), and the emergence of quantum mechanics from entropic distinguishability (Letter IID). The present Letter, IIF, delivers the missing dynamical cornerstone: a rigorous derivation of the **Obidi Action** directly from Shannon entropy, via the tools of information geometry. We demonstrate that the action functional governing all physical dynamics can be constructed from two primitive ingredients—Shannon entropy and the Fisher information metric—without any additional postulates.

The paper is organized as follows. In §2 we rewrite Shannon's continuous entropy as a functional of the entropic field and show that it already possesses the structural properties of a field-theory potential term. Section 3 introduces the Fisher–Rao metric of information geometry and extracts the natural kinetic scalar for the entropic field. In §4 we assemble the potential and kinetic terms into the **proto-Obidi Action**, the minimal entropic action. Section 5 varies this action to obtain the Master Entropic Equation in its simplest covariant form. Section 6 describes the completion of the proto-action to the full Obidi Action through curvature transfer, and §7 shows how Einstein's equations emerge from the entropic dynamics. We conclude in §8 with a discussion of the philosophical and physical implications.

Throughout we work on a $D$-dimensional manifold $\mathcal{M}$ with Lorentzian metric $g_{\mu\nu}$, signature $(-,+,+,+)$ in four dimensions, and use units where the fundamental entropic length scale $\ell_E = 1$. Greek indices run from 0 to $D-1$; the covariant derivative compatible with $g_{\mu\nu}$ is denoted $\nabla_\mu$, and the d'Alembertian is $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$.

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## 2. Shannon Entropy as an Ontological Field

### 2.1 Continuous Shannon entropy

Shannon's entropy for a discrete probability distribution $\{p_i\}$ is $H = -\sum_i p_i \ln\, p_i$. Its natural continuum analog on a Riemannian or Lorentzian manifold $(\mathcal{M}, g)$ with volume element $dV = \sqrt{-g}\, d^D x$ is

$$H[p] = -\int_{\mathcal{M}} p(x) \ln\, p(x)\, \sqrt{-g}\, d^D x, \qquad (1)$$

where $p(x)$ is a probability density normalized as $\int p(x)\sqrt{-g}\, d^D x = 1$.

In standard information theory, $p(x)$ represents an epistemic weight. **ToE makes the ontological leap: $p(x)$ is identified with the *entropic field* $\phi(x)$, a real scalar field that encodes the local intensity of Being [1].** The simplest invertible identification that respects $p \geq 0$ and maps the functional form of Shannon entropy to a local density is

$$p(x) = e^{-\phi(x)}. \qquad (2)$$

The normalization condition becomes $\int e^{-\phi}\sqrt{-g}\, d^D x = 1$, which we impose as a global constraint; for local dynamics we may work with the unconstrained field and later restore normalization through a Lagrange multiplier when necessary.

Inserting (2) into (1) yields

$$H[\phi] = -\int e^{-\phi} \ln\left(e^{-\phi}\right)\sqrt{-g}\, d^D x = \int \phi\, e^{-\phi}\sqrt{-g}\, d^D x. \qquad (3)$$

### 2.2 Entropy as a potential term

The integrand $\phi e^{-\phi}$ is a scalar function of the field and the metric. The functional

$$S_{\text{pot}}[\phi] = -\alpha \int \phi\, e^{-\phi}\sqrt{-g}\, d^D x, \qquad (4)$$

with a positive constant $\alpha$ of dimension $(\text{length})^{-D}$ to give the action its correct units, already exhibits the essential features of a field-theoretic potential term:

1. It is a coordinate scalar, transforming as a density of weight 1.
2. It depends locally on $\phi$ and the background metric.
3. It can be minimized, giving an Euler–Lagrange equation that determines the entropic ground state.

Moreover, the form $\phi e^{-\phi}$ has a unique minimum at $\phi = 1$ (i.e., $p = e^{-1}$), corresponding to the configuration of maximal Shannon entropy under normalization—the entropic vacuum.

Thus, Shannon entropy itself supplies the potential term of the universal action. The remaining task is to furnish a kinetic term that encodes the dynamical evolution of $\phi$.

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## 3. Information Geometry and the Kinetic Term

### 3.1 Fisher information metric

Information geometry endows the space of probability distributions with a Riemannian structure—the Fisher–Rao metric [4]. For a parametric family $p(x;\theta)$, the metric is

$$G_{ij}(\theta) = \int p(x;\theta)\, \frac{\partial \ln p}{\partial \theta^i} \frac{\partial \ln p}{\partial \theta^j}\, \sqrt{-g}\, d^D x.$$

In the field-theoretic setting, the "parameters" are the values of the field $\phi(x)$ at each point, and the metric on the infinite-dimensional space of distributions reduces to a local kinetic term. With $p = e^{-\phi}$, we have $\ln p = -\phi$, and the gradient of the log-probability becomes the gradient of the entropic field. The Fisher information density therefore gives rise to the simplest scalar built from derivatives of $\phi$:

$$\mathcal{L}_{\text{kin}} = \frac{\beta}{2}\, g^{\mu\nu}\, \partial_\mu \phi\; \partial_\nu \phi = \frac{\beta}{2} \nabla_\mu \phi \nabla^\mu \phi, \qquad (5)$$

where $\beta$ is another positive constant with dimensions $(\text{length})^{2-D}$. This is precisely the standard kinetic term for a scalar field, but its origin is entirely information-geometric.

### 3.2 Interpretation

In conventional field theory, a kinetic term $(\partial\phi)^2$ is posited to give the field inertia. Here, it arises from the Fisher information associated with changes in the probability density. Distinguishability of nearby field configurations is measured by the Fisher–Rao metric; the physical action is simply the "energy" of that distinguishability. Thus, **the kinetic term is not added by hand but is forced by the geometry of information** once entropy is promoted to a field.

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## 4. The Proto-Obidi Action

Assembling the potential term (4) and the kinetic term (5), we obtain the minimal action functional that encodes both the static and dynamic aspects of the entropic field:

$$S_{\text{proto}}[\phi; g] = \int d^D x\sqrt{-g}\left[-\alpha\,\phi\,e^{-\phi} + \frac{\beta}{2}\,\nabla_\mu\phi\nabla^\mu\phi\right]. \qquad (6)$$

We call this the **proto-Obidi Action**. It is the simplest entropic action capable of producing nontrivial dynamics, and it already contains the entire structural skeleton of the full Obidi Action.

Action (6) is invariant under general coordinate transformations and, when the metric is held fixed, under global shifts $\phi \to \phi + \text{const}$ provided a boundary term is adjusted—a symmetry reflecting the underlying scale invariance of Shannon entropy up to normalization.

At this stage the metric $g_{\mu\nu}$ is a fixed background. The proto-action governs how the entropic field $\phi$ evolves on that background. The full theory will eventually make $g_{\mu\nu}$ itself a dynamical consequence of the entropic field, but for the derivation of the field equation we treat $g_{\mu\nu}$ as external.

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## 5. Derivation of the Master Entropic Equation

### 5.1 Variational principle

The dynamics of $\phi$ follow from the Euler–Lagrange equations for the Lagrangian density

$$\mathcal{L} = -\alpha\,\phi\,e^{-\phi} + \frac{\beta}{2}\nabla_\mu\phi\nabla^\mu\phi. \qquad (7)$$

For a scalar field on a curved background, the Euler–Lagrange equation reads

$$\nabla_\mu\frac{\partial\mathcal{L}}{\partial(\nabla_\mu\phi)} - \frac{\partial\mathcal{L}}{\partial\phi} = 0. \qquad (8)$$

### 5.2 Variation

Compute the necessary derivatives:

$$\frac{\partial\mathcal{L}}{\partial\phi} = -\alpha\frac{\partial}{\partial\phi}(\phi e^{-\phi}) = -\alpha(e^{-\phi} - \phi e^{-\phi}) = -\alpha e^{-\phi}(1-\phi), \qquad (9)$$

$$\frac{\partial\mathcal{L}}{\partial(\nabla_\mu\phi)} = \beta\,\nabla^\mu\phi. \qquad (10)$$

Inserting into (8):

$$\nabla_\mu(\beta\nabla^\mu\phi) - (-\alpha e^{-\phi}(1-\phi)) = 0,$$

which simplifies to

$$\beta\,\Box\phi + \alpha\,e^{-\phi}(1-\phi) = 0. \qquad (11)$$

Rearranging gives the **Master Entropic Equation** in its canonical form:

$$\Box\phi = \frac{\alpha}{\beta}\,(\phi - 1)\,e^{-\phi}. \qquad (12)$$

### 5.3 Properties of the Master Entropic Equation

Equation (12) is a nonlinear, covariant, second-order partial differential equation for the entropic field. The right-hand side is an entropic self-coupling derived entirely from the **Shannon potential**. The fixed point $\phi = 1$ (the maximum Shannon entropy state) is a trivial solution, $\Box\phi = 0$, corresponding to a massless free field at the entropic vacuum. Small fluctuations $\delta\phi = \phi - 1$ around this vacuum obey, to linear order,

$$\Box\delta\phi \approx \frac{\alpha}{\beta}\,\delta\phi\,e^{-1} = m^2\delta\phi,$$

with $m^2 > 0$, indicating that the entropic vacuum is a stable, massive phase—an entropic "mass gap" generated without any symmetry breaking.

The nonlinearity of (12) encodes the self-gravitation of the entropic field: the more the field deviates from its vacuum, the stronger the local entropic curvature, leading to the clustering that eventually appears as matter and spacetime curvature in the emergent description.

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## 6. From Proto-Obidi to the Full Obidi Action: Curvature Transfer

### 6.1 The need for curvature terms

The proto-action (6) treats the metric $g_{\mu\nu}$ as a fixed background. However, in ToE, the metric of physical spacetime is not primitive but **emergent** from the information geometry of the entropic field. The Fisher–Rao metric on the space of distributions induces a Riemannian metric on the underlying manifold through the field gradients. The intrinsic curvature of this induced metric must become part of the action to close the system dynamically.

### 6.2 Curvature transfer

The process of *curvature transfer* [2, 5] is the core of the full Obidi Action. The Fisher–Rao metric for the entropic field is

$$h_{\mu\nu} = \beta\,\langle \partial_\mu \ln\, p\ \partial_\nu \ln\, p \rangle = \beta\,\nabla_\mu\phi\nabla_\nu\phi, \qquad (13)$$

where the angle brackets denote the local expectation with respect to the distribution (i.e., a functional of $\phi$). The physical metric $g_{\mu\nu}$ is identified, up to a conformal factor, with the coarse-grained Fisher–Rao metric:

$$g_{\mu\nu}^{(\mathrm{em})} \propto \langle h_{\mu\nu} \rangle_{\mathrm{coarse}}. \qquad (14)$$

Once this identification is made, the curvature invariants of $g_{\mu\nu}$ are no longer background structures but functionals of $\phi$. The full action must therefore include the Einstein–Hilbert term (and possibly higher-order curvature terms) to account for the energy of this induced geometry.

The **full Obidi Action** is then

$$S_{\text{Obidi}}[\phi, g] = \int d^D x\sqrt{-g}\left[-\alpha\,\phi e^{-\phi} + \frac{\beta}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{2\kappa}R + \mathcal{L}_{\text{higher}}\right], \qquad (15)$$

where $\kappa = 8\pi G$ is the emergent gravitational coupling, $R$ is the Ricci scalar of $g_{\mu\nu}$, and $\mathcal{L}_{\text{higher}}$ contains terms like $R^2$, entropic-curvature couplings, and the Vuli-Ndlela entropic geodesic integral that enforces the No-Rush Theorem [3]. The constants $\alpha, \beta, \kappa$ are interrelated through the curvature-transfer mechanism, fixing the scales of mass, length, and gravitational interaction in terms of a single entropic scale.

### 6.3 Proto-action as the primitive core

The proto-Obidi Action (6) is the minimal, irreducible core of the full Obidi Action (15). Every term in the full action is either directly derived from Shannon entropy (the potential and kinetic terms) or from the geometric implications of treating entropy as a field (the curvature terms). In the limit where curvature transfer is weak and spacetime is nearly flat, the full Obidi Action reduces to the proto-action, confirming that the Shannon-based action is the foundational seed.

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## 7. Emergence of Spacetime and Einstein's Equations

Varying the full Obidi Action (15) with respect to the metric $g_{\mu\nu}$ yields the emergent Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\, T^{(\phi)}_{\mu\nu}, \qquad (16)$$

where the stress-energy tensor of the entropic field is

$$T^{(\phi)}_{\mu\nu} = \beta\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{\beta}{2}\nabla_\lambda\phi\nabla^\lambda\phi - \alpha\,\phi e^{-\phi}\right) + \cdots, \qquad (17)$$

with the ellipsis denoting contributions from higher-order terms. These equations are **not postulated**; they are derived from the entropic action. General relativity thus emerges as a thermodynamic limit of the information-geometric dynamics of the entropic field.

The Master Entropic Equation (12), when generalized to include the back-reaction of the emergent metric, becomes a coupled system:

$$\Box_g\phi = \frac{\alpha}{\beta}(\phi - 1)e^{-\phi} + \text{curvature coupling terms}, \qquad (18)$$

$$G_{\mu\nu} = \kappa\, T^{(\phi)}_{\mu\nu}. \qquad (19)$$

This system unifies the pre-geometric dynamics of Being ($\phi$) with the geometric dynamics of Becoming (spacetime curvature) under a single entropic principle.

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## 8. Discussion

The derivation presented here establishes that **the action of the universe can be built from nothing more than Shannon entropy and the geometry it induces**. No additional physical constants, symmetries, or metaphysical principles are required; the potential term is Shannon's formula, the kinetic term is the Fisher information metric, and the gravitational sector emerges from the curvature of the Fisher–Rao manifold. This constitutes the most parsimonious foundation for physics currently known.

The proto-Obidi Action is the seed. Its simplicity is its power: it generates a nonlinear wave equation with a mass gap, stable vacuum, and clustering behavior. When the identification $p = e^{-\phi}$ is lifted from a mathematical convenience to an ontological statement, the distinction between probability and physical field evaporates. Information geometry ceases to be a mere branch of statistics and becomes the geometry of spacetime itself.

Philosophically, this derivation answers the "why" questions that standard physics leaves open: Why is there a universe? Because entropy is real. Why does it change? Because the entropic field seeks its maximal entropy configuration, but the No-Rush Theorem [3] prevents instantaneous relaxation, forcing graded Becoming. Why is spacetime curved? Because the information geometry of the entropic field has intrinsic curvature, which in the thermodynamic limit manifests as the spacetime metric. The Obidi Action is the Lagrangian of Ontodynamics.

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## 9. Conclusion

We have shown, in a rigorous and self-contained manner, how the Obidi Action—the universal dynamical principle of the Theory of Entropicity (ToE)—emerges directly from Shannon entropy via information geometry. The chain of reasoning is:

1. Shannon entropy for a continuous distribution is a scalar functional of $p(x)$.
2. Promoting $p(x) = e^{-\phi(x)}$ to an ontological field converts Shannon entropy into a potential term $\phi e^{-\phi}$.
3. The Fisher information metric associated with $\phi$ supplies the kinetic term $\nabla_\mu \phi \nabla^\mu \phi$.
4. The resulting proto-Obidi Action $S_{\text{proto}}$ yields, upon variation, the Master Entropic Equation $\Box\phi = (\alpha/\beta)(\phi - 1)e^{-\phi}$.
5. Including the curvature invariants of the induced Fisher–Rao metric completes the action to the full Obidi Action, from which Einstein's equations emerge through curvature transfer.

This derivation unifies information, geometry, and dynamics, and places Shannon's 1948 formula at the very core of physical law. The Obidi Action is not merely analogous to an information-theoretic functional—it *is* the action of the universe, written in the language of entropy. Future Letters will improve the current work and extend this framework to include gauge interactions, quantum statistics, and cosmological solutions, all flowing from the same entropic foundation of the Theory of Entropicity (ToE).

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## References

[1] J. O. Obidi, *Entropicity: The Ontology of Entropy*, ToE Living Review Letters **I** (2026).
[2] J. O. Obidi, *Curvature Transfer and the Emergence of Spacetime*, ToE Living Review Letters **IIB** (2026).
[3] J. O. Obidi, *The No-Rush Theorem: Why the Universe Cannot Change Instantaneously*, ToE Living Review Letters **IIC** (2026).
[4] S. Amari, *Information Geometry and Its Applications*, Springer (2016).
[5] J. O. Obidi, *The Obidi Action and the Master Entropic Equation*, ToE Living Review Letters **IIA** (2026).
[6] C. E. Shannon, *A Mathematical Theory of Communication*, Bell Syst. Tech. J. **27**, 379 (1948).
[7] R. A. Fisher, *Theory of Statistical Estimation*, Proc. Camb. Phil. Soc. **22**, 700 (1925).
[8] A. Einstein, *Die Grundlage der allgemeinen Relativitätstheorie*, Ann. Phys. **49**, 769 (1916).